GRADUATE COURSE ON ROBUST HYBRID CONTROL SYSTEMS – Homework #2

Problem 1 *(30 points)* Consider a simplified model of the Newton's cradle consisting of a pair of pendulums with mass m_L, m_R as shown in Figure 1.

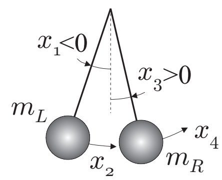

Figure 1: Simplified Newton's cradle.

1.1 Determine the state of the system.

1.2 Define each element of a hybrid system model (C, f, D, g) assuming the following:

- Viscous friction for circular motion.
- Conservation of momentum at impacts.
- Dissipation of energy at impacts.

1.3 Perform the following simulations of the resulting hybrid system for parameters $m_L = 1, m_R = 2$, unitary viscous friction, and a restitution law with parameters such that there is dissipation at impacts:

(a) Plot trajectories as a function of t for pendulums starting at $x_1 = -\pi/4$ and $x_3 = \pi/4$ with zero velocity.

(b) Plot trajectories as a function of t for pendulums starting at $x_1 = 0$ and $x_3 = \pi/4$ with zero velocity.

(c) Plot trajectories as a function of t for pendulums starting at $x_1 = \pi/4$ and $x_3 = \pi/2$ with zero velocity.

Problem 2 *(30 points)* Consider the implementation of a static controller

$$\kappa : \mathbb{R}^n \to \mathbb{R}^m$$

for the continuous-time plant

$$\dot{\xi} = \tilde{f}(\xi, u)$$

in a *digital device*, e.g. computer, microcontroller, digital signal processor, etc. This is depicted in Figure 2, where the controller is interfaced with sample-and-hold devices. The sample-and-hold device that samples the state ξ of the plant is referred to as *sampling device* (or analog-to-digital (A/D) converter), while the sample-and-hold device that stores the output of the controller in between computations is referred to as *hold device* (or digital-to-analog (D/A) converter) which is assumed to be of zero-order type, that is, a zero-order hold (ZOH).

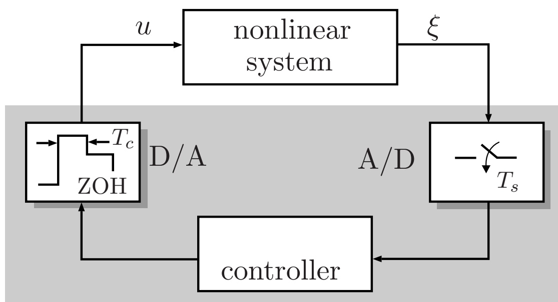

Figure 2: Sample-and-hold control of a nonlinear system.

2.1 Define each element of a hybrid system model (C, f, D, g) and its state assuming the following:

- The computation of the static feedback law takes no time, i.e., is instantaneous.
- The positive constants T_s and T_c are not necessarily equal.

2.2 Explain how the model would change if the computation of the control law takes $\delta > 0$ units of time.

Problem 3 *(30 points)* For the hybrid system $\mathcal{H}$ with data

$$\begin{aligned} C &:= \left\{ x \in \mathbb{R}^2 \ : \ |x| < 1 \right\}, & f(x) &:= \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix} \\ D &:= \left\{ x \in \mathbb{R}^2 \ : \ x_1 = 0, x_2 \in (-\tfrac{1}{2}, 0] \right\}, & g(x) &:= \tfrac{x}{2} \end{aligned}$$

with state $x \in \mathbb{R}^2$,

1. Classify all of its solutions following the definitions in Definition 2.5 and Definition 2.7.
2. Indicate the points in $\overline{C} \cup D$ from which nontrivial solutions exist.
3. Indicate the properties of maximal solutions to $\mathcal{H}$.

You may want to apply Proposition 2.10 in answering these questions.

Problem 4 *(10 points)* For the hybrid system $\mathcal{H}$ that you proposed in Homework 1, answer items 1-3 in Problem 3 above.