**GRADUATE COURSE ON ROBUST HYBRID CONTROL SYSTEMS – Homework #3**

**Problem 1** *(40 points)* For the bouncing ball system given by the hybrid system with

$$f(x) = \begin{bmatrix} x_2 \\ -\gamma \end{bmatrix}, \qquad C := \left\{ x \in \mathbb{R}^2 \ : \ x_1 \geq 0 \ \right\}$$
$$g(x) = \begin{bmatrix} x_1 \\ -ex_2 \end{bmatrix}, \qquad D := \left\{ x \in \mathbb{R}^2 \ : \ x_1 = 0, x_2 \leq 0 \ \right\}$$

where $e \in [0, 1)$ and $\gamma > 0$:

1. Show that the origin $(0, 0)$ is asymptotically stable.
2. In the plane, plot: a) the flow and jump sets, b) level sets of the Lyapunov functions, and c) a solution starting from $(1, 0)$ and a solution starting from $(0, -1)$. Show graphically that the motion of the solutions is such that they go from larger to smaller level sets of the Lyapunov function.

**Problem 2** *(40 points)* Consider the hybrid system with state $x \in \mathbb{R}^2$ and data

$$\begin{aligned} C &:= \{x : x_1 \geq 0\}, & f(x) &:= \begin{bmatrix} \alpha & \omega \\ -\omega & \alpha \end{bmatrix} x \qquad \forall x \in C, \\ D &:= \{x : x_1 = 0, x_2 \leq 0\}, & g(x) &:= -\gamma x \qquad \forall x \in D, \end{aligned}$$

where $\gamma > 0$, $\omega > 0$, and $\alpha \in \mathbb{R}$ are the system parameters.

1. Using the sufficient conditions for Lyapunov stability, find conditions on the system parameters for which the origin of the hybrid system is *uniformly globally pre-asymptotically stable*. Show your work in detail.
2. Confirm your answer to item 1 via simulations.
3. Is the origin *uniformly globally asymptotically stable*? Justify your answer.

**Problem 3** *(20 points)* For the hybrid system $\mathcal{H}$ that you proposed in Homework 1:

1. Define a set $\mathcal{A}$ that is to be stabilized.
2. Study the stability properties of the system, either using the definition of asymptotic stability or using the sufficient conditions in terms of Lyapunov functions.