GRADUATE COURSE ON HYBRID CONTROL SYSTEMS – Homework #1

Suggested reading: First 10 pages of

R. Goebel, R. G. Sanfelice and A. R. Teel. Hybrid Dynamical Systems. IEEE Control Systems Magazine, 2009.

which is available from
`https://hybrid.soe.ucsc.edu/files/preprints/34.pdf`

and Chapter 1 of

R. Goebel, R. G. Sanfelice and A. R. Teel. Hybrid Dynamical Systems: Modeling, Stability, and Robustness, Princeton University Press, 2012

which is available from

`http://press.princeton.edu/chapters/s9759.pdf`

Problem 1 (30 points) Download and install the simulation toolbox available at

`https://hybrid.soe.ucsc.edu/software`

Read through the documentation that comes with it, which also available at

`https://hybrid.soe.ucsc.edu/files/preprints/TR-SCN-HyEQ_Toolbox_v202-10-2014.pdf`

and answer the following:

a) List three features of the simulation toolbox.

b) Describe the main parts of the Lite HyEQ Simulator and draw a flow chart describing its operation.

c) Search online for an alternative way to simulate hybrid equations. Explain two advantages (if any) of such tool when compared to the entire HyEQ toolbox. Explain two disadvantages (if any) of such tool when compared to the entire HyEQ toolbox.

2. (30 points) Use the Lite HyEQ Simulator to simulate the hybrid system capturing the dynamics of a timer with reset to zero from the lectures on Day 1. Use x^* as the parameter that defines the threshold for jumps of the state variable x, in which case the flow set we defined becomes

$$C = [0, x^*]$$

$$D = \{x \in \mathbb{R} \; : \; x = x^* \; \}$$

a) Explain if there are any issues in obtaining appropriate trajectories.

b) Can the state x be arbitrarily initialized to generate a trajectory that represents a timer with resets to zero? Explain.

c) Define new flow and jump sets so that the issues in a) and b) are resolved. Validate it numerically and plot for 10 seconds (and submit) three representative trajectories (choose them wisely) .

3. (40 points) Use the Lite HyEQ Simulator to simulate the hybrid system capturing the dynamics of a ball bouncing on the ground from the lectures on Day 1.

a) Explain if there are any issues in obtaining appropriate trajectories.

b) Can the state x be arbitrarily initialized to generate a trajectory that represents a ball bouncing? Explain.

c) Define a new jump set so that the issue in a) is resolved. Validate it numerically and plot for 20 seconds (and submit) a trajectory starting with unitary height and unitary velocity (both positive). Report any problem you may experience.

d) For the same trajectory in c), plot the energy of the ball as a function of ordinary time. Justify its shape.