GRADUATE COURSE ON HYBRID CONTROL SYSTEMS – Homework #2

Suggested reading: First 40 pages of

R. Goebel, R. G. Sanfelice and A. R. Teel. Hybrid Dynamical Systems. IEEE Control Systems Magazine, 2009.

which is available from

`https://hybrid.soe.ucsc.edu/files/preprints/34.pdf`

Chapter 3 of

R. Goebel, R. G. Sanfelice and A. R. Teel. Hybrid Dynamical Systems: Modeling, Stability, and Robustness, Princeton University Press, 2012

and

[18] R. G. Sanfelice, R. Goebel, and A. R. Teel "Invariance principles for hybrid systems with connections to detectability and asymptotic stability", IEEE Transactions on Automatic Control, vol. 52, no. 12, pp. 2282–2297, 2007. `https://hybrid.soe.ucsc.edu/sites/default/files/preprints/18.pdf`

Problem 1 Consider the hybrid closed-loop system $\mathcal{H}$ with state $x \in \mathbb{R}^2$ given in Problem 4 of Homework 1, namely, with data given by

$$C := \left\{ x \in \mathbb{R}^2 \ : \ x_1 \geq 0 \right\}, \qquad F(x) := \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix} \qquad \forall x \in C$$
$$D := \left\{ x \in \mathbb{R}^2 \ : \ x_1 = 0, x_2 \leq 0 \right\}, \qquad G(x) := -x \qquad \forall x \in D$$

where $\alpha \in \mathbb{R}$.

1. Show that the origin is stable.
2. Is the origin attractive?
3. Replace G by $G(x) = -\alpha x$ where $\alpha \in \mathbb{R}$. Determine the range of values for α such that $\mathcal{A} = \{0\}$ is pre-asymptotically stable. For those values, is $\mathcal{A}$ globally asymptotically stable?

Problem 2 Consider the algorithm in Problem 5 of Homework 1 and suppose that an additional goal is to guarantee that the origin for the plant is global asymptotically stable. Does the algorithm guarantee such a property? If not, sketch an algorithm that, in addition to the requirements in Problem 5 of Homework 1, will achieve global asymptotic stability.

Problem 3 Answer the following questions:

1. Given the hybrid system $\mathcal{H}$ with state $x \in \mathbb{R}$ and data

$$\begin{aligned} C &:= [0,2]\,, & F(x) &:= 3 - x & \forall x \in C \\ D &:= [1,3]\,, & G(x) &:= 0 & \forall x \in D \end{aligned}$$

what is a maximal solution from $\xi = 0$?

2. Given the hybrid system $\mathcal{H}$ with state $x \in \mathbb{R}$ and data

$$\begin{aligned} C &:= \left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right], \qquad F(x) := \begin{cases} 1 - x & \text{if } x \geq \frac{1}{2} \\ -1 - x & \text{if } x \leq -\frac{1}{2} \end{cases} \qquad \forall x \in C \\ D &:= \mathcal{A}, \qquad G(x) := -x \qquad \forall x \in D \end{aligned}$$

where $\mathcal{A} = \{x \in \mathbb{R} \ : \ |x| = 1 \ \}$ is the set $\mathcal{A}$ pre-asymptotically stable? Show it mathematically or provide a counterexample.

Problem 4 Prove the expressions in equations (2.3) and (2.4) in [65].

Problem 5 Consider the hybrid system with state $x \in \mathbb{R}^2$ and data

$$\begin{aligned} C &:= \{x : x_1 \geq 0\}, & f(x) &:= \begin{bmatrix} \alpha & \omega \\ -\omega & \alpha \end{bmatrix} x \qquad \forall x \in C, \\ D &:= \{x : x_1 = 0, x_2 \leq 0\}, & g(x) &:= -\gamma x \qquad \forall x \in D, \end{aligned}$$

where $\gamma > 0$, $\omega > 0$, and $\alpha \in \mathbb{R}$ are the system parameters.

1. Using sufficient conditions for pre-asymptotic stability, find conditions on the system parameters for which the origin of the hybrid system is *globally pre-asymptotically stable*.
2. Confirm your answer to item 1 via simulations.
3. Is the origin *globally asymptotically stable*? Justify your answer.

Problem 6 Select a hybrid system (plant, controller, or closed loop) that you are interested in and perform the following tasks:

1. Explain why is **truly** hybrid.
2. Describe its behavior in general terms.
3. Model it as a hybrid system as learned in class.
4. Pick an appropriate set of parameters for your system and provide trajectories for different initial conditions.
5. Define a closed set that you would like your system to have as pre-asymptotically stable. Argue why one expect this set to be stable and pre-attractive, and if a control algorithm is needed for such a property to hold, sketch a possible algorithm that would accomplish that.