GRADUATE COURSE ON HYBRID CONTROL SYSTEMS – Homework #2

Suggested reading:

[70] Robust Supervisory Control for Uniting Two Output-Feedback Hybrid Controllers with Different Objectives, Sanfelice, R. G., and Prieur C. , Automatica, July, Volume 49, Number 7, pp.1958?1969, (2013)

[33] Supervising a family of hybrid controllers for robust global asymptotic stabilization, Sanfelice, R. G., Teel A. R., and Goebel R. , Proc. 47th IEEE Conference on Decision and Control, pp. 4700?4705, (2008)

[11] A "throw-and-catch" hybrid control strategy for robust global stabilization of nonlinear systems, Sanfelice, R. G., and Teel A. R. , Proc. 26th American Control Conference, pp. 3470?3475, (2007)

[25] Robust global swing-up of the pendubot via hybrid control, O'Flaherty, R., Sanfelice R. G., and Teel A. R. , Proc. 27th American Control Conference, pp.1424?1429, (2008)

[97] An Observer with Measurement-triggered Jumps for Linear Systems with Known Input, Ferrante, F., Gouaisbaut F., Sanfelice R. G., and Tarbouriech S. , Proceedings of the 19th IFAC World Congress, pp.140–145, (2014)

which are available from

`https://hybrid.soe.ucsc.edu/biblio`

Problem 1 (20 points) Show that the closed-loop system defined by the feedback interconnection of a well-posed hybrid plant $\mathcal{H}_P$ and a well-posed hybrid controller $\mathcal{H}_K$ is well posed. Explain how the data of the closed loop has to be defined.

Problem 2 (25 points) Propose a well-posed hybrid controller $\mathcal{H}_K$ that implements an event triggered version of the control law

$$\zeta = \kappa(v)$$

in which the events occur when the norm of the output of the plant has changed a $\lambda > 0$ amount. Determine needed properties on κ and the data of the controller you propose.

Problem 3 (30 points) Design a robust global stabilizer of the upright configuration for the single-link pendulum given by

$$\ddot{\theta} + \frac{\gamma}{\ell} \sin \theta = 0$$

where θ is the angle relative to the vertical (zero at the resting position), $\gamma > 0$ is the gravity constant, and $\ell > 0$ is the pendulum's length. Validate your design numerically.

Problem 4 (25 points) Design a robust global stabilizer of the two-point set

$$\mathcal{A} = \{-z^*\} \cup \{z^*\}, \qquad z^* \in \mathbb{R} \setminus \{0\}$$

for the hybrid plant $\mathcal{H}_P$ with state $z \in \mathbb{R}^2$, input $u \in \mathbb{R}^2$, and data

$$C_P = \left\{ (z,u) \ : \ |z|_{\mathcal{A}} \geq \frac{|z^*|}{2}, u_1 = 1 \right\} \cup \left\{ (z,u) \ : \ |z|_{\mathcal{A}} \geq \frac{|z^*|}{2}, u_1 = -1 \right\}$$

$$F_P(z,u) = \begin{bmatrix} \dfrac{(u_1+1)}{2} u_2 \\ \dfrac{(1-u_1)}{2} u_2 \end{bmatrix} \qquad \forall (x,u) \in C_P$$

$$D_P = \{ (z,u) \ : \ z = 0 \ \}$$

$$G_P(x,u) = \mathcal{A} \qquad \forall (x,u) \in D_P$$