GRADUATE COURSE ON HYBRID CONTROL SYSTEMS – Homework #2

Suggested reading:


which are available from

https://hybrid.soe.ucsc.edu/biblio
Problem 1 (20 points) Show that the closed-loop system defined by the feedback interconnection of a well-posed hybrid plant $\mathcal{H}_P$ and a well-posed hybrid controller $\mathcal{H}_K$ is well posed. Explain how the data of the closed loop has to be defined.
Problem 2 (25 points) Propose a well-posed hybrid controller $\mathcal{H}_K$ that implements an event triggered version of the control law

$$\zeta = \kappa(v)$$

in which the events occur when the norm of the output of the plant has changed a $\lambda > 0$ amount. Determine needed properties on $\kappa$ and the data of the controller you propose.
Problem 3 (30 points) Design a robust global stabilizer of the upright configuration for the single-link pendulum given by

\[ \ddot{\theta} + \frac{\gamma}{\ell} \sin \theta = 0 \]

where \( \theta \) is the angle relative to the vertical (zero at the resting position), \( \gamma > 0 \) is the gravity constant, and \( \ell > 0 \) is the pendulum’s length. Validate your design numerically.
Problem 4 (25 points) Design a robust global stabilizer of the two-point set

\[ \mathcal{A} = \{-z^*\} \cup \{z^*\}, \quad z^* \in \mathbb{R} \setminus \{0\} \]

for the hybrid plant \( \mathcal{H}_P \) with state \( z \in \mathbb{R}^2 \), input \( u \in \mathbb{R}^2 \), and data

\[ C_P = \left\{ (z, u) : |z|_A \geq \frac{|z^*|}{2}, u_1 = 1 \right\} \cup \left\{ (z, u) : |z|_A \geq \frac{|z^*|}{2}, u_1 = -1 \right\} \]

\[ \mathcal{F}_P(z, u) = \begin{bmatrix} \frac{(u_1 + 1)}{2} u_2 \\ \frac{(1 - u_1)}{2} u_2 \end{bmatrix} \quad \forall (x, u) \in C_P \]

\[ \mathcal{D}_P = \{(z, u) : z = 0 \} \]

\[ \mathcal{G}_P(x, u) = \mathcal{A} \quad \forall (x, u) \in \mathcal{D}_P \]