GRADUATE COURSE ON HYBRID CONTROL SYSTEMS – Homework #3

Suggested reading:

[75] R. G. Sanfelice ”On the existence of control Lyapunov functions and state-feedback laws for hybrid systems”, IEEE Transactions on Automatic Control, vol. 58, no. 12, pp. 3242?3248, December, 2013.

[96] R. G. Sanfelice ”Input-Output-to-State Stability Tools for Hybrid Systems and Their Interconnections”, IEEE Transactions on Automatic Control, vol. 59, no. 5, pp. 1360-1366, May, 2014.

[69] R. Naldi, and R. G. Sanfelice ”Passivity-based Control for Hybrid Systems with Applications to Mechanical Systems Exhibiting Impacts”, Automatica, vol. 49, no. 5, pp. 1104?1116, May, 2013.

[52] R. G. Sanfelice ”Interconnections of Hybrid Systems: Some Challenges and Recent Results”, Journal of Nonlinear Systems and Applications, pp. 111?121, 2011.

which are available from

`https://hybrid.soe.ucsc.edu/biblio`

Problem 1 (30 points) Show that the function $V : \mathbb{R}^2 \to \mathbb{R}$ defined for each $x \in \mathbb{R}^2$ as

$$V(x) = x^\top P x, \qquad P = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

is a control Lyapunov function with respect to $\mathcal{A} = \{0\}$ for the hybrid system

$$\mathcal{H} \left\{ \begin{array}{lcl} \left.\begin{array}{lcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -a \sin x_1 - b x_2 + u_c \end{array}\right\} & & (x, u_c) \in C \\ \left.\begin{array}{lcl} x_1^+ & = & x_1 \\ x_2^+ & = & -e x_2 \end{array}\right\} & & x \in D, \end{array} \right. \tag{1}$$

where $a, b \in \mathbb{R}$, $e \in [0, 1)$, $u_c \in \mathbb{R}$,

$$C := \left\{ (x, u_c) \in \mathbb{R}^2 \times \mathbb{R} \ : \ x_1 \leq 0 \ \right\},$$

$$D := \left\{ x \in \mathbb{R}^2 \ : \ x_1 = 0, x_2 \geq 0 \ \right\}$$

Problem 2 (30 points) For the system in Problem 1, design a pointwise minimum-norm feedback that practically asymptotically stabilizes the origin.

Problem 3 (40 points) Rederive the steps in Section 5.2 of [52] that show IOS of the interconnection therein. Does the closed-loop system have the origin asymptotically stable when the inputs are set to zero? If the origin is asymptotically stable, is that property robust to small perturbations?