A Hybrid Adaptive Feedback Law for Robust Obstacle Avoidance and Coordination in Multiple Vehicle Systems

Jorge I. Poveda, Mouhacine Benosman, Andrew R. Teel, Ricardo G. Sanfelice

***Abstract*— This paper presents an adaptive hybrid feedback law designed to robustly steer a group of autonomous vehicles toward the source of an unknown but measurable signal, at the same time that an obstacle is avoided and a prescribed formation is maintained. The hybrid law overcomes the limitations imposed by the topological obstructions induced by the obstacle, which precludes the robust stabilization of the source of the signal by using smooth feedback. The control strategy implements a leader-follower approach, where the followers track, in a coordinated way, the position of the leader.**

I. INTRODUCTION

The problem of source seeking has been extensively studied during the last two decades. A variety of smooth source seeking algorithms for autonomous robots modeled as single integrators and unicycles are presented in [1]. For these types of algorithms, local stability results are developed via classic averaging and singular perturbation theory. Continuous-time algorithms for source seeking are also considered in [2], [3], and [4]. Discrete-time algorithms for source seeking are presented in [5] using finite differences, and in [6] using a Newton-Raphson based algorithm. Stochastic algorithms for source seeking are presented in [7] and [8]. As noted in [4], one of the main challenges in source seeking problems is designing robust feedback laws that are also able to avoid obstacles that interfere with the trajectories of the vehicles. This problem is in general not easy, due to the topological obstructions induced by the obstacles, which preclude the *robust* stabilization of the source of the signal by using a continuous feedback law [9, Thm 6.5]. This is because, when using smooth feedback, it is always possible to find arbitrarily small adversarial signals acting on the states (or vector field), such that a set of initial conditions, possibly of measure zero and not containing the source, can be rendered locally stable. As noted in [4] and [9], in gradient systems the problematic set of initial conditions is usually related to the critical points (different from the source) of the potential function used in the gradient-based feedback law. Motivated by this background, we present in this paper a novel model-free robust hybrid control law, which, in contrast to the existing approaches, overcomes the limitations imposed by the topological obstructions induced by the obstacle, guaranteeing convergence of the vehicle to a neighborhood of the source of the signal from initial conditions in arbitrarily large compact sets. Since our final goal is to steer a group of vehicles toward the unknown source of the signal, we implement a leader-follower control approach, where the leader agent implements the model-free hybrid seeking law, and the followers implement a distributed formation control that operates in a faster time scale.

The remainder of this paper is organized as follows: Section II presents some mathematical preliminaries. Section III formalizes the coordinated source seeking problem. Section IV presents the main results. Section V presents a numerical example, and finally Section VI ends with some conclusions.

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II. PRELIMINARIES

We denote by $\mathbb{R}$ the set of real numbers, and by ($\mathbb{Z}_{\geq 0}$) $\mathbb{Z}$ the set of (nonnegative) integers. Given a compact set $\mathcal{A} \subset \mathbb{R}^n$ and $x \in \mathbb{R}^n$, we use $|x|_{\mathcal{A}} := \inf_{y \in \mathcal{A}} \|x - y\|$, where $\|\cdot\|$ is the Euclidean distance, to denote the distance of x to $\mathcal{A}$. We use $\mathbb{B}$ to denote a closed ball with appropriate dimension, centered at zero, and with unit radius. A set-valued mapping $M : \mathbb{R}^p \rightrightarrows \mathbb{R}^n$ is outer semicontinuous (OSC) at x if for each $(x_i, y_i) \to (x, y) \in \mathbb{R}^p \times \mathbb{R}^n$ satisfying $y_i \in M(x_i)$ for all $i \in \mathbb{Z}_{\geq 0}$, we have $y \in M(x)$. A mapping M is locally bounded (LB) at x if there exists a neighborhood U_x of x such that $M(U_x)$ is bounded. Given a set $K \subset \mathbb{R}^p$, the mapping M is LB and OSC relative to K if the set-valued mapping from $\mathbb{R}^p$ to $\mathbb{R}^n$ defined by $M(x)$ for $x \in K$ and $\emptyset$ for $x \notin K$ is LB and OSC at each $x \in K$. We use $\mathbb{S}^1$ to denote the set $\mathbb{S}^1 := \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 = 1\}$. A directed graph is represented by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is a set of nodes, and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is a set of edges. If $(i, \ell) \in \mathcal{E}$ then node i has access to the information of node ℓ. A directed path is a sequence of nodes such that any pair of consecutive nodes in the sequence is a directed edge of $\mathcal{G}$. A graph $\mathcal{G}$ has a globally reachable node if one of its nodes can be reached from any other node by traversing a directed path. In this paper we will consider *hybrid dynamical systems* (HDS) aligned with the framework of [10]. A HDS with state $x \in \mathbb{R}^n$ is represented as $\mathcal{H} := (C, F, D, G)$, and it is characterized by the inclusions

$$\dot{x} \in F(x) \qquad x \in C \tag{1a}$$

$$x^+ \in G(x) \qquad x \in D, \tag{1b}$$

where the set-valued mappings $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ and $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$, called the flow map and the jump map, respectively, describe the evolution of the state x when it belongs to the flow set C or/and the jump set D, respectively. We always impose the following conditions on the data of the system:

(C1) The sets C and D are closed.

(C2) F is OSC and LB relative to C, and $F(x)$ is convex and nonempty for every $x \in C$.

(C3) G is OSC and LB relative to D, and $G(x)$ is nonempty for every $x \in D$.

Solutions of (1) are indexed by a continuous-time index t, and a discrete-time index j, generating *hybrid time domains*, see [10, Ch.2] for details on the formal definition of solutions to (1). A compact set $\mathcal{A}$ is said to be uniformly globally asymptotically stable (UGAS) for $\mathcal{H}$ if there exists a $\mathcal{KL}$ function β such that any solution x to $\mathcal{H}$ satisfies $|x(t,j)|_{\mathcal{A}} \leq \beta(|x(0,0)|_{\mathcal{A}}, t+j)$ for all $(t,j) \in \text{dom}(x)$. Note that definition of UGAS does not insist that every solution needs to have an unbounded time domain. However, every solution with an unbounded time domain (called *complete*) must converge to $\mathcal{A}$. For a HDS parametrized by a small positive parameter ε, and denoted as $\mathcal{H}_\varepsilon := (C_\varepsilon, F_\varepsilon, D_\varepsilon, G_\varepsilon)$, a compact set $\mathcal{A} \subset \mathbb{R}^n$ is said to be semiglobally practically asymptotically stable (SGP-AS) as $\varepsilon \to 0^+$ if there exists a function $\beta \in \mathcal{KL}$ such that the following holds: for each $\Delta > 0$ and $\nu > 0$ there exists $\varepsilon^* > 0$ such that for each $\varepsilon \in (0, \varepsilon^*)$ each solution x of $\mathcal{H}_\varepsilon$ that satisfies $|x(0,0)|_{\mathcal{A}} \leq \Delta$ also satisfies $|x(t,j)|_{\mathcal{A}} \leq \beta(|x(0,0)|_{\mathcal{A}}, t+j) + \nu$, for all $(t,j) \in \text{dom}(x)$.

III. PROBLEM STATEMENT

We consider a group of N autonomous vehicles, each one modeled as a two-dimensional point mass with dynamics

$$\left.\begin{array}{l} \dot{x}_i = u_{x,i} \\ \dot{y}_i = u_{y,i}, \end{array}\right\}, \quad \forall\, i \in \{1, 2, \ldots, N\}, \tag{2}$$

where $u_{x,i}, u_{y,i} \in \mathbb{R}$ are independent velocity inputs to the vehicle. Without loss of generality we assume that the vehicle $i = 1$ is the leader vehicle, and that the vehicles share information via a directed unweighted time-invariant graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ is the set of nodes representing the N vehicles, and $\mathcal{E}$ is the set of edges representing the communication links between vehicles. We impose the following assumption on this communication graph.

Assumption 3.1: The leader vehicle is a globally reachable node for the graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. □

We assume that there exists an unknown signal J that can be measured by the leader vehicle, and that attains its maximum value J^* at some unknown point $[x^*, y^*]^\top \in \mathbb{R}^2$. For the purpose of analysis we assume that J satisfies the following assumption.

Assumption 3.2: $J : \mathbb{R}^2 \to \mathbb{R}$ is smooth and it has a strict global maximum $[x^*, y^*]^\top \in \mathbb{R}^2$. Moreover, for each $\alpha \in \mathbb{R}$ the set $\{(x,y) : J(x,y) \geq \alpha\}$ is compact, and contains no points where $\nabla J(x,y) = 0$ other than $[x^*, y^*]^\top$. □

To model the coordinated behavior of the followers, let $\Xi := \big\{[x_1^f, y_1^f]^\top,\ [x_2^f, y_2^f]^\top, \ldots, [x_N^f, y_N^f]^\top\big\}$ be a collection of N positions in the plane. Then, we say that the N vehicles satisfy the formation Ξ if their location belongs to the set

$$\begin{aligned} \mathcal{F}_\Xi := \Big\{ & [x^\top, y^\top]^\top \in \mathbb{R}^{2N} : x_i = x_i^f + \zeta_x, \\ & y_i = y_i^f + \zeta_y, \quad \forall\, i \in \mathcal{V},\ [\zeta_x, \zeta_y]^\top \in \mathbb{R}^2 \Big\}. \end{aligned} \tag{3}$$

Finally, we also consider the existence of an obstacle $\mathcal{N} \subset \mathbb{R}^2$, located away from the source of the signal J. We assume that the position of the obstacle $\mathcal{N}$ is known only by the leader vehicle. Based on this, our main goal in this paper is to design a *distributed* and *robust* feedback law that guarantees that the leader will converge to a neighborhood of the unknown point $[x^*, y^*]^\top$ that maximizes J, avoiding the obstacle $\mathcal{N}$, and that the follower agents asymptotically achieve a pre-specified formation Ξ around the leader.

IV. ROBUST ADAPTIVE HYBRID DYNAMICS

In order to solve the source seeking problem with obstacle avoidance and formation control, we design a feedback law where the leader vehicle implements a *robust* hybrid adaptive law, and the followers implement a distributed formation control that tracks the position of the leader. To present the key ideas behind the adaptive law, and to motivate the implementation of a hybrid controller, we start by considering a non-hybrid law for the leader, which solves the source seeking problem when no obstacles exist. After this, we "hybridize" this law in order to solve, in a robust way, the source seeking problem with obstacle avoidance.

A. Smooth Seeking Dynamics: The Obstacle-Free Case

For the case when there are no obstacles, we consider a velocity adaptive control law for the leader vehicle, given by

$$u_{x,1} = a\omega\mu_2 + k\xi_x \tag{4a}$$

$$u_{y,1} = -a\omega\mu_1 + k\xi_y, \tag{4b}$$

where $k := \sigma \cdot \bar{\omega}$, and $[\sigma, \bar{\omega}, a, \omega]^\top \in \mathbb{R}^4_{>0}$. The dynamics of (ξ_x, ξ_y) and μ are given by

$$\dot{\xi}_x = -\bar{\omega}\left(\xi_x - 2a^{-1}J(x,y)\mu_1\right) \tag{5a}$$

$$\dot{\xi}_y = -\bar{\omega}\left(\xi_y - 2a^{-1}J(x,y)\mu_2\right) \tag{5b}$$

$$\left.\begin{array}{lcl} \dot{\mu}_1 & = & \omega\mu_2 \\ \dot{\mu}_2 & = & -\omega\mu_1 \end{array}\right\}, \quad \mu = [\mu_1, \mu_2]^\top \in \mathbb{S}^1. \tag{5c}$$

This feedback law is similar to those considered in [2] and [1, Ch. 2], with the subtle difference that we model the excitation signals μ_1 and μ_2 by means of the time-invariant oscillator (5c). To analyze this control law, and following the ideas in [1, Ch. 2], consider the time-invariant change of variables

$$\tilde{x} = x_1 - a\mu_1, \quad \tilde{y} = y_1 - a\mu_2, \tag{6}$$

and the new time scale $\tilde{\rho} = \bar{\omega}t$. With these new variables, and using the vector notation $\xi = [\xi_x, \xi_y]^\top$ and $\tilde{p} = [\tilde{x}, \tilde{y}]^\top$, the closed-loop system in the $\tilde{\rho}$-time scale has the form

$$\dot{\xi} = -\left(\xi - 2a^{-1}J(\tilde{p} + a\mu)\mu\right) \tag{7a}$$

$$\dot{\tilde{p}} = \sigma\xi \tag{7b}$$

$$\left.\begin{array}{lcl} \frac{\bar{\omega}}{\omega}\dot{\mu}_1 & = & \mu_2 \\ \frac{\bar{\omega}}{\omega}\dot{\mu}_2 & = & -\mu_1 \end{array}\right\}, \quad \mu \in \mathbb{S}^1. \tag{7c}$$

For values of $\frac{\bar{\omega}}{\omega}$ sufficiently small we can analyze system (7) based on averaging results for nonlinear systems e.g., [11]. The average system is obtained by averaging the dynamics (7a)-(7b) along the solutions of the oscillator (7c). Since the dynamics (7b) do not explicitly depend on the state μ, the averaging step affects only the dynamics (7a). As in [12], to obtain the average system, we perform a Taylor series expansion of $J(\cdot)$ around $\tilde{p}+a\mu$, obtaining $J(\tilde{p}+a\mu) = J(\tilde{p})+a\,\mu^\top\nabla J(\tilde{p})+e_r$, where the term e_r is of order $O(a^2)$. The following lemma is instrumental to analyze system (7) via averaging theory.

Lemma 4.1: Every solution of (7c) satisfies $\int_0^{\frac{2\pi\bar{\omega}}{\omega}}\mu_i(t)dt = 0$, $\frac{\omega}{2\pi\bar{\omega}}\int_0^{\frac{2\pi\bar{\omega}}{\omega}}\mu_i(t)^2dt = \frac{1}{2}$, and $\int_0^{\frac{2\pi\bar{\omega}}{\omega}}\mu_i(t)\mu_j(t)dt = 0$, for all $i\neq j$. $\diamondsuit$

Using the Taylor expansion of $J(\tilde{p}+a\mu)$ in (7), averaging the right hand side of (7a) over one period of the periodic signals μ, and using Lemma 4.1, we obtain the average system in the $\tilde{\rho}$-time scale

$$\dot{\xi}^A = -(\xi^A - \nabla J(\tilde{p}^A) - e_r) \tag{8a}$$

$$\dot{\tilde{p}}^A = \sigma\cdot\xi^A \tag{8b}$$

where e_r is now of order $O(a)$. Considering the new time scale $\alpha=\sigma\tilde{\rho}$, system (8) is in singular perturbation form for values of $\sigma>0$ sufficiently small, with dynamics (8a) acting as fast dynamics, and dynamics (8b) acting as slow dynamics. The stability of the fast dynamics is analyzed by setting $\sigma=0$ in (8b), which freezes the state $\tilde{p}$. By linearity of (8a) this system has the equilibrium point $\nabla J(\tilde{p}^A)+e_r$ exponentially stable. To obtain the slow dynamics, we replace ξ^A by $\nabla J(\tilde{p}^A)+e_r$ in (8b), obtaining the following slow dynamics in the α-time scale and with new state $\tilde{z}\in\mathbb{R}^2$

$$\dot{\tilde{z}} = \nabla J(\tilde{z}) + e_r. \tag{9}$$

Therefore, under an appropriate tunning of the parameters $(\frac{\bar{\omega}}{\omega},\sigma,a)$, the feedback law given by equations (4) and (5), applied to the vehicle (2), approximates, on average and in the slowest time scale, a gradient system. By setting $e_r=0$ and under Assumption 3.2, this unperturbed gradient system guarantees uniform global asymptotic stability of the unique equilibrium point of J via the Lyapunov function $J(\tilde{z}^*)-J(\tilde{z})$, where $\tilde{z}^*:=[x^*,y^*]^\top$. Using the continuity of $\nabla J(\cdot)$ and the uniform asymptotic stability of the unperturbed system, by [10, Lemma 7.20], we get that this same point is SGP-AS as $a\to0^+$ for system (9). Having established a semiglobal practical stability result for the reduced average system (9), the main theorems in [11, Thm. 1] or [12, Thm. 1] can be used to guarantee a semiglobal practical asymptotic stability result for the original closed-loop system (7) with respect to the parameters $\left(\frac{\bar{\omega}}{\omega},\sigma,a\right)$. In light of the change of variables (6), the previous argument implies that the feedback law (4)-(5) can be tuned to guarantee convergence of the vehicle to any ϵ-neighborhood of $[x^*,y^*]^\top$ from any compact set K of initial conditions.

B. Hybrid Seeking Dynamics for Robust Obstacle Avoidance

The results of the previous section show that, provided Assumption 3.2 is satisfied, the feedback law (4) can be tuned to guarantee robust convergence to a neighborhood of $[x^*,y^*]^\top$ by generating solutions that approximate those of (9). However, the direct application of this same feedback law for the case when there are obstacles in the state space may be problematic, even when potential functions are added to J to "push" the vehicles away from the obstacles. To see this, consider Figure 1 where the state space has been divided in three sets $\mathcal{M}_1$, $\mathcal{M}_2$, and $\mathcal{K}$, and consider a controller that generates a closed-loop time-invariant system of the form $\dot{\tilde{z}}=f(\tilde{z})$, with $\tilde{z}(0)=z_0$, where $f(\cdot)$ is assumed to be locally bounded, and where for all $z_0\in\mathbb{R}^2$ there exists at least one (Carathéodory) complete solution. Due to the topological properties of the problem, there exists a curve $\mathcal{M}$ such that for initial conditions on each side of $\mathcal{M}$, the trajectories of the system approach the set $\mathcal{K}$ either from above the obstacle or from below it. Because of this, it is possible to find arbitrarily small signals $e(t)$ acting on the states of the system (or on the vector field), such that some of the trajectories of the closed-loop system will remain in a neighborhood of the line $\mathcal{M}$, and will not converge to the set $\mathcal{K}$. The following assumption and proposition, corresponding to [9, Assump 6.4 and Thm. 6.5], establish this fact.

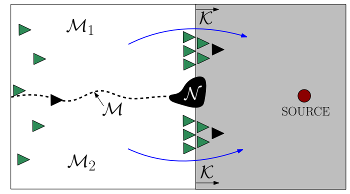

Fig. 1: A group of vehicles following a leader who aims to converge to the source under the presence of the obstacle $\mathcal{N}$.

Assumption 4.1: There exists a $T>0$ such that for each $\tilde{z}_0\in\mathcal{M}$ and each $\rho>0$ there exist points $\tilde{z}_1(0),\tilde{z}_2(0)\in\{\tilde{z}_0\}+\rho\mathbb{B}$, for which there exist (Carathéodory) solutions $\tilde{z}_1$ and $\tilde{z}_2$, respectively, satisfying $\tilde{z}_1(t)\in\mathcal{M}_1\backslash\mathcal{M}$ and $\tilde{z}_2(t)\in\mathcal{M}_2\backslash\mathcal{M}$ for all $t\in[0,T]$. $\square$

Proposition 4.2: [9, Thm 6.5] Suppose that Assumption 4.1 holds. Then for every positive constants ε, ρ', ρ'', and every $\tilde{z}_0\in\mathcal{M}+\varepsilon\mathbb{B}$ such that $\tilde{z}_0+\rho'\mathbb{B}\subset\mathbb{R}^2\backslash\mathcal{N}$ and $\tilde{z}_0+\rho''\mathbb{B}\subset(\mathcal{M}_1\cup\mathcal{M}_2)$ there exist a piecewise constant function $e:\mathrm{dom}(e)\to\varepsilon\mathbb{B}$ and a (Carathéodory) solution $z:\mathrm{dom}(z)\to\mathbb{R}^2\backslash\mathcal{N}$ to $\dot{\tilde{z}}=f(\tilde{z}+e(t))$ such that $\tilde{z}(t)\in(\mathcal{M}+\varepsilon\mathbb{B})\cap(\mathcal{M}_1\cup\mathcal{M}_2)\cap(z_0+\rho'\mathbb{B})$ for all $t\in[0,T')$ for some $T'\in(T^*,\infty]$, where $\mathrm{dom}\;\tilde{z}=\mathrm{dom}\;\tilde{e}$, $T^*=\min\{\rho',\rho''\}m^{-1}$, and $m=\sup\{1+|f(\eta)|:\eta\in z_0+\max\{\rho',\rho''\}\mathbb{B}\}$. $\diamondsuit$

In order to address this issue and to guarantee that the stability properties of the feedback law are not lost under arbitrarily small adversarial signals $e(t)$, we propose to follow the ideas in [13], and to modify the dynamics (4) by partitioning the state space and adding a switching state $q\in\{1,2\}$. Then, our resulting feedback law is hybrid and model-free by nature, and it is based on a *mode-dependent localization function* J_q defined as

$$J_q(x_1,y_1) := -J(x_1,y_1) + B\left(d_q(x_1,y_1)\right), \tag{10}$$

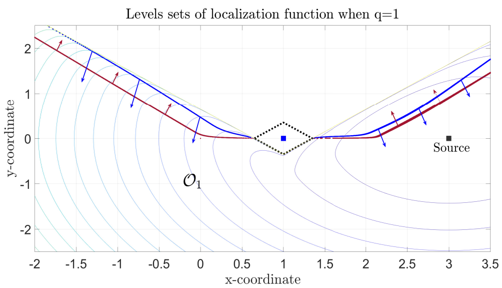

Fig. 2: Space $\mathcal{O}_1$. The blue arrows indicate the flow set and the red arrows indicate the jump set.

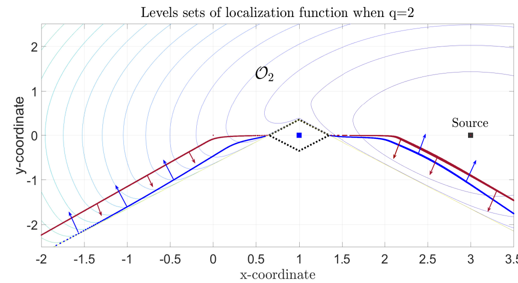

Fig. 3: Space $\mathcal{O}_2$. The blue arrows indicate the flow set and the red arrows indicate the jump set.

where $d_q(x_1, y_1) = \left|[x_1, y_1]^\top\right|^2_{\mathbb{R}^2\backslash\mathcal{O}_q}$. The function $|\cdot|^2_{\mathbb{R}^2\backslash\mathcal{O}_q}$ maps a position $[x_1, y_1]^\top \in \mathbb{R}^2$ to the squared valued of its distance to the set $\mathbb{R}^2\backslash\mathcal{O}_q$, and $B(\cdot)$ is a barrier function defined as follows

$$B(z) = \begin{cases} (z-\rho)^2 \log\left(\frac{1}{z}\right), & \text{if } z \in [0, \rho] \\ 0, & \text{if } z > \rho, \end{cases} \tag{11}$$

with $\rho \in (0, 1]$ being a tunable parameter to be selected sufficiently small. The sets $\mathcal{O}_1$ and $\mathcal{O}_2$ are constructed as shown in Figures 2 and 3. Namely, we construct a box centered around the obstacle $\mathcal{N}$, with tunable height h, and we project the adjacent sides of the box to divide the space in two parts. Figures 2 and 3 also show the level sets of J_q over $\mathcal{O}_q$, for $q \in \{1, 2\}$. Note that $\mathcal{O}_1 \cup \mathcal{O}_2$ covers $\mathbb{R}^2$ except for the box that includes the obstacle. Also, note that under this construction the function (10) is smooth for each q.

To define the set of points where the leader switches the state q, let $p = [x_1, y_1]^\top$, and let $\mu > 1, \lambda \in (0, \mu - 1)$. Using the localization function J_q (10), we define the sets

$$C_J := \left\{(p, q) \in \mathbb{R}^2 \times \{1, 2\} : J_q(p) \leq \mu J_{3-q}(p)\right\}, \tag{12a}$$

$$D_J := \left\{(p, q) \in \mathbb{R}^2 \times \{1, 2\} : J_q(p) \geq (\mu - \lambda) J_{3-q}(p)\right\}. \tag{12b}$$

The blue line and blue arrows in Figures 2 and 3 indicate the points in $\mathcal{O}_q$ that also belong to the flow set C_J, while the red line and the red arrows indicate the points in $\mathcal{O}_q$ that also belong to the jump set D_J. Note that since $(\mu - \lambda) > 1$ the sets C_J and D_J always overlap. The switching rule for q is then given by the mapping

$$Q(q) := 3 - q, \tag{13}$$

and by modifying the dynamics (7) we obtain the model-free hybrid feedback law with flow dynamics

$$\left.\begin{array}{rcl} \dot{\xi}_x &=& -\bar{\omega}\left(\xi_x - 2a^{-1} \cdot J_q(x, y) \cdot \mu_1\right) \\ \dot{\xi}_y &=& -\bar{\omega}\left(\xi_y - 2a^{-1} \cdot J_q(x, y) \cdot \mu_2\right) \end{array}\right\}, \quad \xi \in \mathbb{R}^2 \tag{14a}$$

$$\left.\begin{array}{rcl} \dot{x}_1 &=& a\omega\mu_2 - k\xi_x \\ \dot{y}_1 &=& -a\omega\mu_1 - k\xi_y \\ \dot{q} &=& 0 \end{array}\right\}, \quad (p, q) \in C_J \tag{14b}$$

$$\left.\begin{array}{rcl} \dot{\mu}_1 &=& \omega\mu_2 \\ \dot{\mu}_2 &=& -\omega\mu_1 \end{array}\right\}, \quad \mu \in \mathbb{S}^1. \tag{14c}$$

and jump dynamics

$$\xi_x^+ = \xi_x, \quad \xi_y^+ = \xi_y, \qquad (\xi_x, \xi_y) \in \mathbb{R}^2 \tag{15a}$$

$$x_1^+ = x_1, \quad y_1^+ = y_1, \quad q^+ = Q(q), \quad (p, q) \in D_J \tag{15b}$$

$$\mu_1^+ = \mu_1, \quad \mu_2^+ = \mu_2, \qquad \mu \in \mathbb{S}^1. \tag{15c}$$

The main idea behind the hybrid feedback law (14)-(15) is as follows: The vehicle continuously measures the source signal $J(x, y)$, and at the same time calculates the term $B(d_q(x, y))$ for both $q = 1$ and $q = 2$, obtaining the values $J_1(x(t), y(t))$ and $J_2(x(t), y(t))$ in (10) at each $t \geq 0$. Whenever $q(t) = 1$ and $J_1(t) \geq (\mu - \lambda) \cdot J_2(t)$, the state q is update to $q^+ = 2$, and the signal used in (14a) is changed from J_1 to J_2. Whenever $q(t) = 2$ and $J_2(t) \geq (\mu - \lambda) \cdot J_1(t)$, the state q is update to $q^+ = 1$, and the signal used in (14a) is changed from J_2 to J_1. By implementing this switching feedback law the leader vehicle will always implement -on average- a gradient ascent law over the set $\mathcal{O}_q$, where no problematic set $\mathcal{M}$ arises. The parameter $\mu > 1$ is used to avoid recurrent jumps, while the parameter λ is used to inflate the jump set such that existence of solutions is guaranteed under small perturbations on the state or vector field.

C. Analysis of Hybrid Adaptive Seeking Dynamics

To analyze the hybrid adaptive law (14)-(15) we follow a similar path as in the analysis of system (4)-(5), this time using averaging and singular perturbation results for hybrid dynamical systems and hybrid extremum seeking control, i.e., [14], [15], [16]. First, we restrict the ξ-dynamics to a compact set $\tilde{\lambda}\mathbb{B} \subset \mathbb{R}^2$, where the constant $\tilde{\lambda} \in \mathbb{R}_{>0}$ can be selected arbitrarily large to model any complete solution of interest of the system. Then, applying the change of variable (6) to system (14)-(15) we obtain a hybrid system in vectorial form with flow map in the $\tilde{\rho}$-time scale given by

$$\dot{\xi} = -\bar{\omega}\left(\xi - 2a^{-1} J_q(\tilde{p} + a\mu)\mu\right), \qquad \xi \in \tilde{\lambda}\mathbb{B} \tag{16a}$$

$$\dot{\tilde{p}} = -\sigma\xi, \qquad \dot{q} = 0, \qquad (\tilde{p} + a\mu, q) \in C_J \tag{16b}$$

$$\frac{\bar{\omega}}{\omega}\dot{\mu}_1 = \mu_2, \quad \frac{\bar{\omega}}{\omega}\dot{\mu}_2 = -\mu_1, \qquad \mu \in \mathbb{S}^1, \tag{16c}$$

and jump map

$$\xi^+ = \xi, \qquad \xi \in \lambda\mathbb{B} \tag{17a}$$

$$\tilde{p}^+ = \tilde{p}, \qquad q^+ = Q(q), \quad (\tilde{p} + a\mu, q) \in D_J \tag{17b}$$

$$\mu_1^+ = \mu_1, \quad \mu_2^+ = \mu_2, \qquad \mu \in \mathbb{S}^1. \tag{17c}$$

For this system we will consider the additive term $a\mu$ acting on the equations (16b) and (17b) as a small perturbation acting on the position state $\tilde{p}$. Thus the stability analysis will be based on a *nominal* system where this perturbation is set to zero, using later robustness principles to establish the stability properties for the original perturbed system (16)-(17). Based on this idea, and following the same procedure of Section IV-A, for small values of $\frac{\bar{\omega}}{\omega}$, we obtain the average system with flow map in the α-time scale given by

$$\begin{aligned} \sigma\dot{\xi}^A &= -\left(\xi^A - \nabla J_q(\tilde{p}^A) - e_r\right), & \xi^A \in \tilde{\lambda}\mathbb{B} \\ \dot{\tilde{p}}^A &= -\xi^A, \quad \dot{q}^A = 0, & (\tilde{p}^A, q^A) \in C_J \end{aligned}$$

and jump map given by

$$\begin{aligned} \xi^{A+} &= \xi^A, & \xi^A \in \lambda\mathbb{B} \\ \tilde{p}^{A+} &= \tilde{p}^A, \quad q^{A+} = Q(q^A), & (\tilde{p}^A, q^A) \in D_J. \end{aligned}$$

For values of σ sufficiently small, this HDS is a singularly-perturbed HDS [15], with ξ-dynamics acting as fast dynamics. The reduced or "slow" system is obtained to be the hybrid system with new state $(\tilde{z}, q) \in \mathbb{R}^2 \times \{1, 2\}$, flows in the α-time scale given by

$$\dot{\tilde{z}} = -\nabla J_q(\tilde{z}) + e_r, \quad \dot{q} = 0, \quad (\tilde{z}, q) \in C_J, \tag{18}$$

and jumps given by

$$\tilde{z}^+ = \tilde{z}, \quad q^+ = Q(q), \quad (\tilde{z}, q) \in D_J. \tag{19}$$

The following proposition follows directly by [13, Thm. 4.4], [10, Thm. 7.14], and the fact that e_r in (18) is of order $\mathcal{O}(a)$.

Proposition 4.3: Let $\mu > 1$ and $\lambda \in (0, \mu - 1)$, consider the HDS (18)-(19) and suppose that Assumption 3.2 holds with minimum $\tilde{z}^* \in \mathbb{R}^2$. There exists a $\rho \in (0, 1]$ such that this point is SGP-AS as $a \to 0^+$. Moreover, for a sufficiently small a and a sufficiently small perturbation $e(t)$ acting on the state $\tilde{z}$, there exists a complete solution from every point in the set $\{\tilde{z}\} \times \{1, 2\}$ that is away from the obstacle. ♢

Having established a stability result for the average-reduced hybrid system (18)-(19), we can now obtain a stability result for the original system (14)-(15), which corresponds to the first main result of this paper. The proof follows by the results in [16] and the change of variable (6).

Theorem 4.4: Let $\mu > 1$ and $\lambda \in (0, \mu - 1)$ and suppose that Assumption 3.2 holds with maximizer $\tilde{p}^* = [\tilde{x}^*, \tilde{y}^*]^\top$. Then, there exists a $\rho > 0$ such that for each compact set $K \subset \mathbb{R}^2$ such that $\tilde{p}^* \in \text{int}(K)$ there exists a $\tilde{\lambda} > 0$ such that for each $\varepsilon \in \mathbb{R}_{\geq 0}$ there exists a a^* such that for each $a \in (0, a^*]$ there exists a σ^* such that for each $\sigma \in (0, \sigma^*)$ there exists a $\frac{\bar{\omega}}{\omega}^* \in \mathbb{R}_{>0}$ such that for each $\frac{\bar{\omega}}{\omega} \in (0, \frac{\bar{\omega}}{\omega}^*]$ there exists a T^* such that $\tilde{p}(t, j) \to \tilde{p}^* + \varepsilon\mathbb{B}$ for all $t + j \geq T^*$ for the HDS (16)-(17) with flow set $\tilde{\lambda}\mathbb{B} \times C_J \cap (K \times \{1, 2\}) \times \mathbb{S}^1$ and jump set $\tilde{\lambda}\mathbb{B} \times D_J \cap (K \times \{1, 2\}) \times \mathbb{S}^1$. ♢

D. Followers Dynamics

To guarantee that the followers achieve a formation Ξ, we consider the following formation control law

$$\left.\begin{aligned} u_{x,i} &= -\beta \textstyle\sum_{j \in \mathcal{N}_i} \left(x_i - x_j - x_i^f + x_j^f\right) \\ u_{y,i} &= -\beta \textstyle\sum_{j \in \mathcal{N}_i} \left(y_i - y_j - y_i^f + y_j^f\right), \end{aligned}\right\}, \tag{20}$$

for all $i \in \{2, \ldots, N\}$, where $\beta \in \mathbb{R}_{>0}$ is a tunable parameter. The stability analysis of the follower's dynamics ignores the obstacle and assumes that the position of the leader vehicle is fixed, i.e., $\dot{x}_1 = 0$ and $\dot{y}_1 = 0$. The next lemma, which follows directly by the results in [17], establishes asymptotic convergence toward the desired formation, parametrized by the position of the leader.

Lemma 4.5: Suppose that Assumptions 3.1 hold. Consider the system comprised of the follower dynamics (20) and the leader dynamics $\dot{x}_1 = 0$ and $\dot{y}_1 = 0$ with $[x_1(0), y_1(0)]^\top \in \mathcal{O}_1 \cup \mathcal{O}_2$. Then, every complete solution of this system converges exponentially fast to the point $p_x^* = x^f + \mathbf{1}_N(x_1(0) - x_1^f)$, $p_y^* = y^f + \mathbf{1}_N(y_1(0) - y_1^f)$.

E. Closed-loop System

We now consider the closed-loop system given by the leader vehicle with hybrid feedback law (14)-(15) and the follower vehicles with feedback law (20) for all $i \in \{2, \ldots, N\}$. The following theorem corresponds to the second result of this paper. The proof follows by Lemma 4.5 and results in singular perturbation theory for HDS [15].

Theorem 4.6: Suppose that Assumptions 3.1 and 3.2 hold, and consider the adaptive hybrid feedback law (14)-(15) for the leader vehicle, and the consensus feedback law (20) for the follower agents. Then, the results of Theorem 4.4 hold for the leader vehicle, and for each $\epsilon > 0$ and compact set of initial conditions $K \subset \mathbb{R}^{2N}$ of the vehicles, the parameter $\beta > 0$ can be selected sufficiently small such that any trajectory of the group of followers that avoids the obstacle will converge to the set $\Big(\{x^f + \mathbf{1}_{N-1}(x^* - x_1^f)\} \times \{y^f + \mathbf{1}_{N-1}(y^* - y_1^f)\}\Big) + \epsilon\mathbb{B}$ in finite time. ♢

Theorem 4.6 establishes semiglobal and robust model-free convergence of the leader to the source of the signal J, as well as formation realization for the trajectories of the followers, provided they avoid the obstacle. However, even though the leader will always avoid the obstacle from any initial condition, there is no guarantee that the followers implementing (20) would also avoid the obstacles from all possible initial conditions. Nevertheless, under the hybrid feedback law (14)-(15) for the leader, and the linear feedback law (20) for the followers, the avoidance property for the followers can be obtained if one considers the set of initial conditions $\mathcal{X}_p \subset \mathbb{R}^{2(N-1)}$ satisfying that if p is the initial position of the leader, $\dot{p} = 0$, and $\mathcal{R}(\mathcal{X}_p)$ is the reachable set of (20) from $\mathcal{X}_p$, there exists an $\epsilon > 0$ such that the intersection of $\mathcal{R}(\mathcal{X}_p) + \epsilon\mathbb{B}$ and the obstacle is empty. In this case one could implement the formation control in a faster time scale by selecting β sufficiently large. The previous observation simply suggests the necessity of using a *hybrid formation control* law if one is interested in achieving both objectives: *global* (or semiglobal) *robust* obstacle avoidance *and* robust formation control. The design of such hybrid formation control laws is currently an active research direction.

V. NUMERICAL EXAMPLES

We apply the results of the previous section to a group of 6 vehicles aiming to achieve formation around the source of

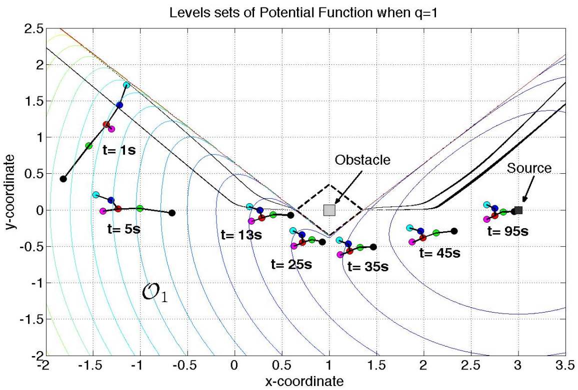

Fig. 4: Evolution of the vehicles over the level sets of J_1.

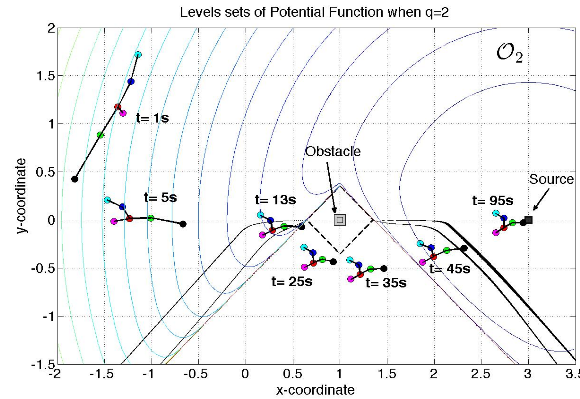

Fig. 5: Evolution of the vehicles over the level sets of J_2.

a signal J, which can be sensed by the leader agent, with an obstacle located at the point $[1, 0]^\top$. For the purpose of simulation we assume that this signal has a quadratic form $J = \frac{1}{2}(x_1 - 3)^2 + \frac{1}{2}y_1^2$. We emulate the situation where the 6 robots are initially located at the entrance of a room, and where the source of the signal J is only know to be located at the other side of the room, with the obstacle $\mathcal{N}$ located between the entrance of the room and the source of the signal J. The parameters of the controller are selected as $h = 0.5$, $\rho = 0.4$, $\lambda = 0.09$, $\mu = 1.1$, $a = 0.01$, $\bar{\omega} = 1$, $k = 1$, and $\beta = 4$. The desired formation is characterized by the set $\Xi = \{(-2,0.5),(-2,-0.5),(-1.13,0),(0,0),(-2.86,1),(-1.13,-1)\}$. Figures 4 and 5 show the position of the vehicles at 7 different time instants, over the virtual level sets of J_q, for $q \in \{1, 2\}$. After approximately 5 seconds the follower agents have achieved the desired formation behind the leader agent (represented by the black dot). The leader implements the hybrid feedback law initially with $q = 2$, and at approximately 9 seconds it enters the jump set shown in Figure 5, and updates its logic mode state as $q^+ = 1$, flowing now along the level sets shown in Figure 4, until convergence is achieved to the source of the signal. Since the box around the obstacle is constructed sufficiently large compared to the size of the formation, the followers also avoid the obstacle by achieving the formation around the leader in a faster time scale and by maintaining the required formation while the leader slowly moves toward the unknown source of the signal.

VI. CONCLUSIONS

In this paper we presented a robust adaptive hybrid feedback law for a group of vehicles seeking for the source of an unknown signal J, and aiming to achieve a desired formation. The feedback law implementes a switching state q that is switched based on a geometric construction around the obstacles position. By implementing this hybrid law, no problematic set of measure zero of initial conditions emerges. Using a consensus based law, the follower vehicles can also achieve a desired formation parameterized by the position of the leader from initial conditions whose inflated reachable set does not include the obstacles position.

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