Challenges in Set-Valued Model-Predictive Control

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ABSTRACT

In this abstract we describe a framework for computationally-aware computing through set-valued model predictive control. Model-predictive control (MPC) can enable multi-objective optimization in real-time, though it depends on accurate models through which future state values can be predicted. This abstract improves upon existing MPC approaches in that it considers the state to be a set (rather than a singleton in the state), allowing the trajectories to be given by a sequence of sets. The framework is beneficial for physical systems control where the uncertainty in future projection can be attributed to both model error, and environmental or sensor uncertainty, thus providing guarantees of performance, robustly. We provide an overview of the framework, and include discussion for its advantages.

CCS CONCEPTS

• **Computer systems organization → Robotic control**; **Embedded and cyber-physical systems**.

KEYWORDS

Model Predictive Control, Cyber-physical systems, Robust control

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1 INTRODUCTION AND MOTIVATION

Model-Predictive Control (MPC) has successfully been applied to many online control problems where system constraints and goals can be expressed as a cost function. Through the use of a predictive model that simulates a candidate input stream over a fixed time horizon, an optimizer can select the input sequence that results in an acceptably low (and in some cases, the lowest) cost according to the function definition. Key limitations in the application of MPC emerge when using nonlinear models for which closed-form optimized solutions cannot be pre-computed. In these cases, there are competing concerns of achieving accurate predictions of future behavior, while simultaneously performing rapid iteration in selecting inputs to optimize those predictions. Previous work has demonstrated the merits of leveraging trade-offs in accuracy vs. computation speed at different regions of the state space. In particular, work by members of the proposing team has demonstrated how simplifications in the system model can be accommodated [8] if the execution frequency of the MPC system can be sufficiently increased according to metrics for switching criteria between accurate (but slow-to-compute) and less-accurate (but fast-to-compute) models [7].

Despite these advancements in switching control that are responsive to computation time, the approaches still rely on computing single trajectories based on traditional dynamical models. In the case of systems with uncertainty, the formulation of predictive controllers requires models which are capable to capture system dynamics, constraints, and also system uncertainty, which may lead to a spread in possible trajectories. This variability can be represented by using a set based formulation.

In this abstract, we explore theoretical and practical challenges for representing sets of behaviors as set dynamical systems [6]. If successful, this new approach could result in an ability to operate within the limitations of models, providing confidence in safe behaviors at runtime that are typically associated with verification.

2 FRAMEWORK

2.1 Model-Predictive Control

With a discrete-time control system described as $x^+ = \widetilde{g}(x, u)$ where $x \in \mathbb{R}^n$ represents the state of the system (and x^+ the next state), and $u \in \mathbb{R}^m$ the control input, let $\widetilde{g}(\cdot)$ define its true behavior. A common approach to implement a discretized nonlinear MPC is to use a model $x^+ = g(x, u)$ with $g(x, u) \approx \widetilde{g}(x, u)$, in this way approximating the discrete-time control system for the purposes of prediction. Using this approximation model, the MPC framework traditionally consists of defining a cost functional

$$\mathcal{J}(\mathbf{x}, \mathbf{u}) = \sum_{j=1}^{J-1} \ell(x_j, u_j) + V(x_J) \tag{1}$$

where $\ell(\cdot)$ represents the stage cost and $V(\cdot)$ represents the terminal cost, with the argument $\mathbf{x}$ of $\mathcal{J}$ denoting a given state trajectory and $\mathbf{u}$ a control input signal to $x_{k+1} = g(x_k, u_k)$. With this cost

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functional, given an initial condition $x_\circ$, the MPC problem is defined as selecting that input stream via the optimization

$$\min_{\mathbf{u}\in U \,:\, x^+=g(x,u), x(0)=x_\circ} \mathcal{J}(\mathbf{x},\mathbf{u}) \tag{2}$$

Various approaches such as Tube-based MPC [3] have provided insights into how to keep a single trajectory within a set that defines a tube of uncertainty. The approach we explore goes beyond the uncertainty of a single trajectory (as defined by a set or tube) to rather consider how a set's shape can be predicted into future timesteps using set dynamics.

2.2 Set-Valued Dynamics

Based on previous work [6], we express set-valued dynamics as

$$X^+ = G(X, U) \tag{3}$$

where X is the set-valued state, and U the set-valued input; we describe $G : \mathcal{P}(\mathbb{R}^n) \times \mathcal{P}(\mathbb{R}^m) \rightrightarrows \mathcal{P}(\mathbb{R}^n)$, where $\mathcal{P}(S)$ represents the powerset of S, as a set-valued map that defines the evolution of the set-valued states.

With this basis (leaving additional formalisms and properties of the above dynamics to previous work in [1, 4, 5]) we describe set-valued analogs to the dynamics given in (1) and (2).

A set-based cost functional is defined as

$$\mathcal{J}(\mathbf{X},\mathbf{U}) = \sum_{j=1}^{J-1} \ell(X_j, U_j) + V_f(X_j) \tag{4}$$

with each X_j and U_j representing the set-values at that time step, $\ell : \mathcal{P}_C(\mathbb{R}^n) \times \mathcal{P}(\mathbb{R}^m) \to \mathbb{R}_{\geq 0}$ is the stage cost, and $V_f : \mathcal{P}_C(\mathbb{R}^n) \to \mathbb{R}_{\geq 0}$, with $\mathcal{P}_C(S)$ the set of all compact nonempty subsets of S, is the terminal cost. A sequence of compact nonempty sets $\mathbf{X}$ defines the state trajectory, and a sequence of closed nonempty sets $\mathbf{U}$ is the input.

If the calculation of the previous cost, along with the propagation of the set-valued state using (3) are possible, then we pose the analogous set-valued MPC problem as

$$\min_{(\mathbf{X},\mathbf{U})\in \hat{S}(X_0)} \mathcal{J}(\mathbf{X},\mathbf{U}) \tag{5}$$

with the initial state defined as the set X_0, and $\hat{S}(X_0)$ expressing the set of all solution pairs $(\mathbf{X},\mathbf{U})$ with initial set X_0.

3 OPPORTUNITIES AND CHALLENGES

If it is possible to describe a set-valued dynamics analog to MPC, then there are several key advantages to the approach.

- Execution of set-valued MPC could provide runtime guarantees that are typically associated only with system models that have undergone verification analysis;
- The output solution pairs $(\mathbf{X},\mathbf{U})$ could be compared to constraints that should be avoided (collision, etc.);
- Approximations $G_1(\cdot)$, $G_2(\cdot)$, etc., to the ideal system's set-valued dynamical model $G(\cdot)$ that may be faster to compute can be compared to an ideal model's solution pairs—which could help to determine what level of accuracy is required to execute the system at runtime;
- Uncertainties in system execution and set-dynamics can be lumped in prediction of future states; and
- Captured dynamical data could be fitted to sets providing new opportunities for set-based system identification.

The framework for Set-Valued MPC has similar structure to traditional MPC, but in order to execute it, theoretical advancements are needed.

Distance or error: In order to optimize a solution pair $(\mathbf{X},\mathbf{U})$ in comparison to a previous candidate, it is necessary to understand what makes one candidate input set $\mathbf{U}$ lower cost than another one—and in turn how to translate that change in cost to a change in inputs that would result in lower cost.

Set-Valued MPC Properties: Under what conditions will a set-valued MPC problem be stable? Can invariance be shown? What is the computational cost of solving MPC for trajectories given in terms of sets? Can these set-valued trajectories be approximated for faster computing, and what would be the accuracy associated?

Approaches to demonstrate these properties, as well as definitions and formalisms that support the metrics and properties required to explore concepts of stability, continue to see advancement [2]. Results include simulations that demonstrate the analysis concepts, and set-dynamical maps in the domain of car-like robot control to validate the approaches.

4 CONCLUSIONS

Set-Valued MPC could provide a unique way in which families of system behaviors (rather than individual trajectories) can be simulated and explored. Promising approaches to theoretical advancements in this area continue to be developed, and can be compared to the properties of physical systems through families of inputs.

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REFERENCES

[1] J. Crowley, Y. Zeleke, B. Altın, and R. G. Sanfelice. 2019. Set-Based Predictive Control for Collision Detection and Evasion. In *2019 IEEE 15th International Conference on Automation Science and Engineering (CASE)*. 541–546. https://doi.org/10.1109/COASE.2019.8842963

[2] Rafal Goebel and Saša V Raković. 2019. Set-valued and Lyapunov methods for MPC. In *Handbook of Model Predictive Control*. Springer, 53–73.

[3] S. V. Raković and Q. Cheng. 2013. Homothetic tube MPC for constrained linear difference inclusions. In *2013 25th Chinese Control and Decision Conference (CCDC)*. 754–761. https://doi.org/10.1109/CCDC.2013.6561023

[4] N. Risso and R. G. Sanfelice. 2016. Detectability and Invariance Properties for Set Dynamical Systems. *IFAC-PapersOnLine* 49, 18 (2016), 1030–1035. https://doi.org/10.1016/j.ifacol.2016.10.303 10th IFAC Symposium on Nonlinear Control Systems NOLCOS 2016.

[5] N. Risso and R. G. Sanfelice. 2017. Sufficient conditions for asymptotic stability and feedback control of set dynamical systems. In *2017 American Control Conference (ACC)*. 1923–1928. https://doi.org/10.23919/ACC.2017.7963233

[6] R. G. Sanfelice. 2014. Asymptotic properties of solutions to set dynamical systems. In *53rd IEEE Conference on Decision and Control*. 2287–2292. https://doi.org/10.1109/CDC.2014.7039736

[7] K. Zhang, J. Sprinkle, and R. G. Sanfelice. 2015. A Hybrid Model Predictive Controller for Path Planning and Path Following. In *International Conference on Cyber-Physical Systems (ICCPS)*. ACM, ACM, Seattle, WA, 139–148. https://doi.org/10.1145/2735960.2735966

[8] K. Zhang, J. Sprinkle, and R. G. Sanfelice. 2016. Computationally-Aware Switching Criteria for Hybrid Model Predictive Control Of Cyber-Physical Systems. *IEEE Transactions on Automation Science and Engineering* 13 (04/2016 2016), 479–490. https://doi.org/10.1109/TASE.2016.2523341