Towards Hybrid Model Predictive Control for Computationally Aware Satellite Applications

CHRISTOPHER PETERSEN and SEAN PHILLIPS, Air Force Research Laboratory, USA
DAWN HUSTIG-SCHULTZ and RICARDO SANFELICE, University of California Santa Cruz, USA

Model Predictive Control (MPC) is an optimal control method that is attractive for safe, efficient and goal based satellite operation. However, current satellite systems have limited computation and thus standard MPC approaches are limited. To overcome this, we propose a hybrid dynamical systems framework to encompass both satellite and optimizer dynamics. This enables a practical analysis of MPC and allows for user trade off between feasibility and optimality via tuneable parameters while retaining asymptotic stability.

Additional Key Words and Phrases: feedback control, model predictive control, hybrid systems

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1 INTRODUCTION AND MOTIVATION

Model Predictive Control (MPC) is an optimization-based, feedback technique that has found success in chemical processing [5, 13], automobiles [6, 9], unmanned aerial vehicles [14, 16], and satellites [4, 10–12, 17–21]. Two common assumptions for MPC are (A1) that the optimization needed for feedback is solved on time scales sufficiently faster than the plant dynamics and (A2) the optimization is solved exactly before the feedback action is enacted. This decouples the computational complexity from the feedback, but is not practical for general applications as optimization takes physical time and, if too slow, the plant may "miss" when it must actuate. This work takes the approach of combining the plant dynamics and optimization into one hybrid dynamical system framework, allowing for a more practical analysis of MPC and enabling a mechanism for user trade off between computation, feasibility, and optimality.

There are limited MPC solutions that consider the explicit coupling of the plant-optimization and its associated computational restrictions. Some works treat computation as a fixed-time delay [3], removing direct consideration of the optimization. The article [9] takes a similar approach to this work by using a dynamical system formulation of the optimization in the MPC loop, but computation time is not directly taken into consideration. For restrictions that prevent convergence to the exact optimum, articles such as [7, 15] discuss using suboptimal MPC policies, but do not jointly consider the computation time. This hybrid MPC solution herein is unique since it allows for a simultaneous investigation of computation time, feasibility, optimality, and stability.

In this work, we will focus on satellite operations. The field is diverse in its applications, given its use in orbital transfers [17], station-keeping [19], attitude control [12], in-space robotics [18], and rendezvous, proximity operations and docking (RPOD) [4, 10, 11, 20, 21]. Many satellite processors are radiation hardened, which increases their size, weight, and power while simultaneously reducing available processing (at least 5-10 times that of a standard computer) and memory (most times less than 256 MB). Thus "instantaneous computation", i.e assumption (A1), cannot apply. For these applications, it is then sometimes preferred to have a solution that is feasible but not quite optimal rather than take the extra time to obtain optimality, i.e. violating assumption (A2). This work will enable such a philosophy. Through the appropriate selection of parameters, the hybrid MPC framework will, in general, enable users to prioritize feasibility or optimality, all while retaining asymptotic stability for operations. In addition, and in particular settings, this framework will allow near optimal performance when implemented in real-world platforms.

2 PROBLEM FORMULATION AND MODELS

Consider two disjoint layers of dynamics. The first layer encompasses the dynamics of the physical system, namely, the satellite. The second layer encompasses the optimization mechanics. MPC connects the two by first using the lower layer to optimize, over a finite window, an objective function subject to constraints. A portion of that optimal solution is then given to the upper layer to enact and the process is repeated. Under proper assumptions, this produces a stabilizing feedback action that yields an ever evolving optimal path. This is the standard MPC formulation. However, we in this work do not assume (A1) and (A2) previously as stated. The overall problem is as follows:

Problem: Asymptotically stabilize the satellite to a desired set point with practical optimality taking into account the computational constraints onboard the satellite.

Addressing the above leads to a compromise, either computation at the price of optimality or vis, versa. In our proposed approach, this is exemplified via a switching condition that halts the evaluation of the MPC algorithm and allows the control input to execute first available input. Naturally, this coupling between the continuous-time dynamics of the satellite model with the dynamics of the optimizer leads to modeling the system as leveraging the hybrid inclusion framework in [2]. The separate models are now defined, with the proposed hybrid approach defined thereafter.

Upper Layer-Satellite Model: The satellite dynamics are specialized in this work to the controlled docking maneuver of one satellite to a free-floating, uncontrolled satellite. In most applications and under reasonable assumptions, this is analogous to stabilizing a

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linear-time invariant system with inputs to the origin (i.e. Clohessy-Wiltshire equations [1]). We start with this model and then begin to relax assumptions, increasing complexity, and thereby highlighting the trade offs in computation.

Lower Layer-Optimization: At its core, MPC recursively enacts an optimization solver to find the solution to a well-defined optimal control problem (OCP). All solvers require some aspect of iteration, and as such can be treated as their own system. In this work, the solver is chosen as a dynamical system reformulation of gradient descent with constraints [8]. Solving for the optimum then entails only propagating the mathematical model capturing the optimization algorithm until it approaches a neighborhood of the optimum. The rate of convergence for this model is tuneable via a modeling parameter. This parameter is related to the magnitude of the "Newton-step" or "velocity state" of the solver, and our preliminary work suggests that under proper assumptions the rate is exponential.

3 HYBRID MPC APPROACH

The hybrid model combines the satellite dynamics with the continuous-gradient optimization solver, see Figure 1. These two layers ideally run in parallel, with the plant input held constant in sample-and-hold between subsequent solvings. To enable the hybrid model, logic and timer states are employed to indicate when the plant input is updated and when to begin or stop the optimization solver.

The stopping condition is of particular importance, as it dictates when the optimization stops and the satellite enacts the control. In this work, it is based on the change in the cost. When this change is smaller than the ratio between a small tolerance and the Newton-step, then the solution is close to the optimum and can (though does not have to) be executed. This small tolerance is another tuneable parameter that relates to the trade off in computation.

The switch between the optimizer and the plant is governed by a supervisor. When the solution is close to the optimum – as determined by the stopping condition – and a timer has reached a fixed threshold, the supervisor updates the logic state, updates the plant input with the first step of the solver input, and the timer is reset to zero. Conversely, when the plant state is far from the optimum and the timer reaches the fixed threshold, then the supervisor updates the logic state, starts the optimization, and resets the timer to zero, after which the optimization starts again.

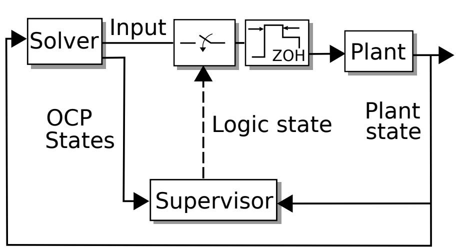

Fig. 1. The proposed hybrid approach to MPC taking into consideration computational constraints onboard the plant.

This hybrid MPC approach is unique as it allows for the analysis between computation costs and optimality via two parameters; namely, the optimizer rate and the stopping threshold. In particular, when the optimizer rate is large, the rate of convergence increases, and it is hypothesized that this is at a cost of transients that may not be close to the optimum or even feasible. In contrast, when stopping threshold is large, the distance to the optimum may be greater at the end of the process, but it is hypothesized that feasibility within the optimizer can be retained easier. Irregardless of this choice, this method will be demonstrated asymptotically stable.

4 ADVANTAGES AND RESEARCH QUESTIONS

From an applied perspective, this approach has several implications for satellites. A specific example is in the computational requirements for docking and how they vary as a function of distance. When distances between the chief and deputy are large, time between maneuvers are on the order of a quarter of an orbit (for Low Earth Orbit, this is once ever 15 minutes, for Geostationary Orbit this is once every 6 hours). Computation times can thus be longer and maneuvers can focus on optimality. When close in for docking however, maneuvers will need to happen at most every minute, if not sooner, so feasibility is a focus and computation needs to be as rapid as possible. This trade off for computation/optimality becomes then a function of mission set and distance, which can be exploited in this hybrid MPC formulation.

If successful, our proposed approach for MPC that is aware of computations provides the following unique advantages:

- *Integration of computational models in the feedback control loop*: This is typically not considered during control design. A framework that allows modeling computing and physics elements would lead to MPC algorithms that take into consideration physical and computational constraints.
- *Computational optimality and stability certificates*: These would allow straightforward consideration of hardware platform limits when implementing the control algorithm.
- *Identifiable computation permits*: These would determine the computational needs and limitations for the actual platform to implement the MPC algorithm.
- *Varying computation for different objectives*: Requirements for computation could vary from objective to objective on the same system using the same MPC algorithm.

At the same time, the approach challenges the standard practice of feedback control design and demands new tools for analysis, design, and simulation. In particular,

- *Models of optimization schemes and stopping conditions*: These need to be modeled mathematically in finite-dimension. Techniques to capture optimization algorithms using mathematical models and are amenable to the control theory are needed.
- *Hardware key parameters*: Platforms that run the algorithms need to be systematically identified from the physical system and incorporated in the models of computing. Algorithms to automatically identify such hardware features are mandatory.
- *Entire system, numerical simulation engines*: These environments need to be developed to efficiently provide numerical solutions that represent the behavior of the actual system.

To address these points and materialize the proposed framework, elements from control theory, optimization, and computing hardware have to be synergistically combined.

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