Exploiting Invariance Properties to Certify Always and Eventually Signal Temporal Logic Operators for Hybrid Dynamical Systems

Hyejin Han and Ricardo G. Sanfelice

Abstract— **In this paper, semantics and characterizations of signal temporal logic formulas for hybrid dynamical systems are presented. Hybrid dynamical systems are given in terms of constrained differential and difference inclusions, which, respectively, capture the continuous evolution and the instantaneous events exhibited by solutions. For such systems, the always and eventually operator of signal temporal logic are studied and characterizations in terms of dynamical properties of hybrid systems are presented – in particular, using invariance and finite-time attractivity properties. Sufficient conditions that guarantee the satisfaction of a signal temporal logic formula for a given system through the satisfaction of an untimed formula for an appropriately defined new system are introduced. Specifically, it is shown that satisfying an (untimed) temporal logic formula involving until operators suffices to certify always and eventually signal temporal logic formulas for hybrid systems.**

I. INTRODUCTION

Complex specifications for dynamical and control systems can be efficiently formulated using temporal logic [1], [2]. In fact, temporal logic permits specifying properties for solutions (or traces) that relate to reaching or avoiding a set, both over a finite and an infinite horizon. The variant of temporal logic proposed in [3] permits to specify properties of continuous-time signals that are defined over ordinary time. Such a logic, known as *signal temporal logic* (STL), is suitable for the validation of statements involving logic and temporal operators over finite time horizons.

The original formulation of STL in [3] is inspired by the work in [4], where *metric interval temporal logic* (MITL) is introduced as a "temporal language that constrains the time difference between events only with finite, yet arbitrary, precision." Though originally introduced for continuous-time signals, the satisfaction and certification of STL specifications has been considered for discrete time and hybrid signals. Among the contributions in the literature that are most related to the work in this paper are the results for the certification of STL for hybrid systems modeled as hybrid automata using reachable sets in [5], where reachset temporal logic (RTL) is introduced. More recently, and also for hybrid automata, a tool for the quantification of robustness in the satisfaction of STL formulas that builds from the results in [6], [7] using falsification and SMT solvers is proposed in [8], while a symbolic model checking algorithm that is "refutationally complete" for general STL properties of bounded signals is proposed in [9].

In this paper, a temporal logic formulation for the certification of properties of *hybrid signals* is proposed. With a semantics that builds from the early work in [3], we propose to certify a particular fragment of STL – specifically, always and eventually STL operators – by recasting the dependency on (hybrid) time of STL as the problem of certifying an untimed formula for a properly defined new hybrid system. The hybrid signals considered in this paper are solutions to hybrid dynamical systems given in a general framework that, as a difference to the work in [5], [8], [9], allows for solutions that may end prematurely (e.g, deadlock), are not bounded (e.g., exhibit finite escape times), or are Zeno – or, more extremely, only evolve discretely. More precisely, inspired by the ideas in [10]–[12], we formulate in Section III the semantics of STL for a broad class of hybrid dynamical systems modeled by differential inclusions and difference inclusions with state constraints, as in [13].

As a difference from the STL formulations in the literature, the temporal operators proposed in this paper involve *hybrid time domains*, which is a time structure that captures continuous evolution (or flow) over ordinary time and jumps (when the state changes instantaneously) using a discrete counter. After formulating the semantics of this (hybrid) signal temporal logic, in Section IV we characterize STL formulas involving the always and eventually operators in terms of forward invariance and finite-time convergence properties. Specifically, we introduce sufficient conditions that guarantee the satisfaction of STL formulas through the satisfaction of LTL formulas involving until operators (LTL for the same class of hybrid dynamical systems was formulated in [14]). An important difference between our results and those in the literature is that, in particular, relative to [5], our results do not require the computation of the reachable set and, relative to [8], symbolic abstractions are not involved. Conveniently, with the approach proposed in this paper, the sufficient conditions recently proposed in [15] to guarantee the satisfaction of LTL formulas involving until operators can be employed to certify STL specifications, for which the price to pay is finding Lyapunov or barrier functions. An example illustrates the concepts and results.

II. OUTLINE OF THE PROPOSED APPROACH

In this paper, we present characterizations of STL formulas using the always and eventually operators for hybrid signals generated by hybrid dynamical systems. For easy of exposi-

H. Han and R. G. Sanfelice are with the Department of Electrical and Computer Engineering, University of California, Santa Cruz, CA 95064.
Research partially supported by the NSF Grants no. ECS-1710621, CNS-2039054, and CNS-2111688, by the AFOSR Grants no. FA9550-19-1-0169, FA9550-20-1-0238, and FA9550-23-1-0145, by the AFRL Grant nos. FA8651-22-1-0017 and FA8651-23-1-0004, and by ARO Grant no. W911NF- 20-1-0253.

tion, we outline our approach for continuous-time systems[1]

$$\dot{x} \in F(x) \qquad\qquad x \in C \subset \mathbb{R}^n \tag{1}$$

which we denote as $\mathcal{H}_f = (C, F)$, and then present the semantics and results for the more complex case of hybrid systems. To reason about these formulas, we consider a new system which implicitly encodes the timing conditions. For (1), the state of this new system consists of a pair (x, τ), where x is the state of the original system and τ is a new state indicating the continuous-time evolution. Then, we define new atomic propositions over the states of the new system based on the interval $\mathcal{I} \subset \mathbb{R}_{\geq 0} := [0, \infty)$ associated with the specification. As we show in this paper, this approach reduces the problem of verifying a timed STL property in the original system to an untimed one for the new system.

Proceeding this way, we outline the proposed approach to characterize specific STL operators. Formally defined in Section III for the case of hybrid signals, given an atomic proposition p and a connected interval $\mathcal{I} \subset \mathbb{R}_{\geq 0}$, the always operator over $\mathcal{I}$ is denoted $\Box_{\mathcal{I}} p$. The formula $f = \Box_{\mathcal{I}} p$ is satisfied for a solution ϕ to (1) at $t = 0$ if $p(\phi(t)) = 1$ for all $t \in \mathcal{I} \cap \operatorname{dom} \phi$. To certify this property, we define the set of points that satisfy p as

$$P := \{x \in C : p(x) = 1\} \tag{2}$$

and the set $\mathcal{I} \subset \mathbb{R}_{\geq 0}$ as

$$\mathcal{I} := [T_{\min}, T_{\max}], \tag{3}$$

where $T_{\min} \geq 0$ and $T_{\max} \geq T_{\min}$. Then, a solution ϕ to (1) satisfies $f = \Box_{\mathcal{I}} p$ at $t = 0$ if $\phi(t) \in P$ for all $t \in \mathcal{I} \cap \operatorname{dom} \phi$.

To characterize the behavior of solutions ϕ to (1) while $t \in \mathcal{I}$, the proposed approach introduces the new system mentioned above, which is denoted $\mathcal{H}_{f,\tau}$, as follows. The system $\mathcal{H}_{f,\tau}$ has state $(x, \tau) \in \mathbb{R}^n \times \mathbb{R}_{\geq 0}$ and dynamics

$$\mathcal{H}_{f,\tau}: \quad \dot{x} \in F(x), \ \dot{\tau} = 1 \qquad\qquad (x, \tau) \in C \times \mathbb{R}_{\geq 0}. \tag{4}$$

The state component τ acts as a timer. Note that for each solution $\psi = (\varphi, \tau)$ to $\mathcal{H}_{f,\tau}$ with $\tau(0) = 0$, the component φ is a solution to (1). We notice that to satisfy $\Box_{\mathcal{I}} p$ for each solution ϕ to (1) at $t = 0$, each solution ψ to $\mathcal{H}_{f,\tau}$ starting from[2] $\mathbb{R}^n \times \{0\}$ must remain in $P \times [T_{\min}, T_{\max}]$ for all $t \in [T_{\min}, T_{\max}]$. In fact, due to τ evolving as a timer, if each solution to $\mathcal{H}_{f,\tau}$ starting from $\mathbb{R}^n \times \{0\}$ stays in $P \times [T_{\min}, T_{\max}]$ for each $t \in \mathcal{I}$, then each solution to (1) stays in P for each $t \in \mathcal{I}$. Hence, the satisfaction of $\Box_{\mathcal{I}} p$ for each solution to (1) at $t = 0$ is assured by guaranteeing the following properties for each solution to $\mathcal{H}_{f,\tau}$ from $\mathbb{R}^n \times \{0\}$:

- Each solution ψ stays in $\mathbb{R}^n \times [0, T_{\min})$ until reaching $P \times [T_{\min}, T_{\max}]$ at some $t \in \operatorname{dom} \psi$ (we encode this property in a proposition denoted p_a) – by construction of $\mathcal{H}_{f,\tau}$, this property holds for free if $T_{\min} \in \operatorname{dom} \psi$.
- Once a solution ψ reaches $P \times [T_{\min}, T_{\max}]$, ψ stays in $P \times [T_{\min}, T_{\max}]$ (which we capture by proposition p_b) until reaching $\mathbb{R}^n \times (T_{\max}, \infty)$ (which we capture by proposition p_c) – this property requires establishing that the component φ of ψ remains in P at least for $T_{\max} - T_{\min}$ seconds after reaching it.

As shown in this paper, these properties can be guaranteed by certifying an untimed formula for $\mathcal{H}_{f,\tau}$.

Similar observations can be made for the STL eventually operator, which is denoted $\Diamond_{\mathcal{I}} p$. The formula $f = \Diamond_{\mathcal{I}} p$ is satisfied for a solution ϕ to (1) at $t = 0$ if $p(\phi(t)) = 1$ for some $t \in \mathcal{I} \cap \operatorname{dom} \phi$. The next example employs this operator and further illustrates the proposed approach.

Example 2.1 (robotic manipulation)**.** *Consider an end effector interacting with a surface located at the origin for position. Denoting its position by x_1 and its velocity by x_2, a model capturing the evolution of $x := (x_1, x_2)$ under the effect of a proportional-derivative continuous-time controller is given by*

$$\dot{x}_1 = x_2, \quad \dot{x}_2 \in F_2(x) \tag{5}$$

where $F_2 : \mathbb{R}^2 \rightrightarrows \mathbb{R}^2$ is a set-valued map capturing the contact force and control action, given by [16, Section 3.2.1 and Prop. 7]

$$F_2(x) := \begin{cases} -k_p x_1 - k_d x_2 & \text{if } x_1 < 0 \\ -\xi - k_d x_2, \ \xi \in \overline{\text{con}}\{0, \, b_c x_2\} & \text{if } x_1 = 0 \\ -k_p x_1 - b_c x_2 - k_d x_2 & \text{if } x_1 > 0 \end{cases}$$

where k_p and k_d are tunable gains, $k_c > 0$ and $b_c > 0$ are, respectively, the elastic and damping coefficients of the compliant contact model, and $\overline{\text{con}}$ denotes the closed convex hull operation. It can be shown that, given k_c and b_c, there exist choices for k_p and k_d such that every maximal solution to (5) *converges to zero asymptotically; see [16, Proposition 7]. Consider the atomic proposition defined as*

$$p(x) = 1 \ \text{ if } x_1 \leq 0, x_2 \geq 0, \quad p(x) = 0 \text{ otherwise.} \tag{6}$$

This proposition captures the requirement of the end effector making contact with the surface. The associated system $\mathcal{H}_{f,\tau}$ defined in (4) *has F defined as $F(x) := (x_2, F_2(x))$ for each $x \in \mathbb{R}^2$. Given an interval $\mathcal{I}$ as in* (3)*, the formula $f = \Diamond_{\mathcal{I}} p$ specifies the property of the end effector making contact with the surface in finite time. To certify it, every solution $\psi = (\varphi, \tau)$ to $\mathcal{H}_{f,\tau}$ with $\tau(0) = 0$ has to satisfy $(\varphi(t), \tau(t)) \in P \times \mathcal{I}$ for some $t \in \operatorname{dom} \psi \cap \mathcal{I}$ ($\neq \emptyset$). This finite-time convergence property to a point in $P \times \mathcal{I}$ can be certified by analyzing $\mathcal{H}_{f,\tau}$. This example is revisited in Example 4.5, where the model in* (5) *is augmented by the addition of jumps capturing collisions with the surface.*

III. Signal Temporal Logic for Hybrid Dynamical Systems

In this section, inspired by the ideas in [10]–[12] for continuous-time and discrete-time systems, we define the semantics of STL for the broad class of hybrid systems $\mathcal{H} = (C, F, D, G)$ on $\mathbb{R}^n$ described as follows [13]:

$$\begin{aligned} \dot{x} &\in F(x) \qquad && x \in C \\ x^+ &\in G(x) \qquad && x \in D \end{aligned} \tag{7}$$

[1]A solution $\phi : \operatorname{dom} \phi \to \mathbb{R}^n$ to (1) is given by a locally absolutely continuous function $t \mapsto \phi(t)$ satisfying (1) as indicated in the notion of solution to $\mathcal{H}$ in Section III. Note that in this case, since (1) is a continuous-time system, $\operatorname{dom} \phi \subset \mathbb{R}_{\geq 0}$ and we only use t as time parameter.

[2]Later, we include a set of initial states.

where $x \in \mathbb{R}^n$ is the state variable, $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ denotes the flow map capturing the continuous dynamics on the flow set C, and $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ defines the jump map capturing the discrete dynamics on the jump set D.

A solution ϕ to $\mathcal{H}$ is parameterized by $(t, j) \in \mathbb{R}_{\geq 0} \times \mathbb{N}$, where t is the ordinary time variable, j is the discrete jump variable, $\mathbb{R}_{\geq 0} := [0, \infty)$, and $\mathbb{N} := \{0, 1, \ldots\}$. The domain of ϕ, denoted $\operatorname{dom}\phi \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$, is a hybrid time domain if for every $(T, J) \in \operatorname{dom}\phi$, the set $\operatorname{dom}\phi \cap ([0, T] \times \{0, 1, \ldots, J\})$ can be written as the union of sets $\bigcup_{j=0}^{J}([t_j, t_{j+1}] \times \{j\})$, for a time sequence $0 = t_0 \leq t_1 \leq t_2 \leq \ldots \leq t_{J+1}$. The t_j's with $j > 0$ define the time instants when the state of the hybrid system jumps and j counts the number of jumps. A solution is given by a hybrid arc. A function $\phi : \operatorname{dom}\phi \to \mathbb{R}^n$ is a hybrid arc if $\operatorname{dom}\phi$ is a hybrid time domain and if for each $j \in \mathbb{N}$, the function $t \mapsto \phi(t, j)$ is locally absolutely continuous on the interval $I^j = \{t : (t, j) \in \operatorname{dom}\phi\}$. A hybrid arc ϕ is a solution to $\mathcal{H} = (C, F, D, G)$ if $\phi(0, 0) \in \overline{C} \cup D$; for each $j \in \mathbb{N}$ such that I^j has nonempty interior (the interior of I^j is denoted as int I^j), $\phi(t, j) \in C$ for each $t \in \operatorname{int} I^j$ and $\dot{\phi}(t, j) \in F(\phi(t, j))$ for almost all $t \in I^j$; for each $(t, j) \in \operatorname{dom}\phi$ such that $(t, j+1) \in \operatorname{dom}\phi$, $\phi(t, j) \in D$ and $\phi(t, j+1) \in G(\phi(t, j))$. A solution to $\mathcal{H}$ is called maximal if it cannot be further extended. Finally, it is said to be complete if its domain is unbounded. Given a set X, $\mathcal{S}_{\mathcal{H}}(X)$ denotes the set of maximal solutions to $\mathcal{H}$ starting from $x \in X$. See [13] for more details.

For a given hybrid system $\mathcal{H}$ as in (7), we define operators and specify properties of $\mathcal{H}$ with STL formulas. First, we introduce atomic propositions.

Definition 3.1 (Atomic Proposition)**.** *An atomic proposition p is a statement on the system state x. A proposition p is treated as a single-valued function of x: given x, $p(x)$ is either* `True` *(1 or $\top$) or* `False` *(0 or $\perp$).*

In the following, the syntax of an STL formula f is defined recursively as follows:

$$f ::= p \,|\, \neg f \,|\, f \vee g \,|\, f \,\mathcal{U}_{\mathcal{I}} g, \tag{8}$$

where p is an atomic proposition, and f and g are STL formulas. The operators $\neg, \vee, \mathcal{U}$ are the *negation*, *disjunction*, *until* operator, respectively – note that we consider both strong and weak versions of $\mathcal{U}_{\mathcal{I}}$, which are denoted $\mathcal{U}_{s,\mathcal{I}}$ and $\mathcal{U}_{w,\mathcal{I}}$, respectively. One can also define operators other than the ones that are used for constructing the grammar. Given the operators *negation* and *disjunction*, the operators *conjunction* ($\wedge$), *implication* ($\Rightarrow$), *equivalency* ($\Leftrightarrow$) are defined as $f \wedge g = \neg(\neg f \vee \neg g)$, $f \Rightarrow g = \neg f \vee g$, $f \Leftrightarrow g = (f \Rightarrow g) \wedge (g \Rightarrow f)$, respectively. Furthermore, the operators *eventually* ($\Diamond$) and *always* ($\Box$) are defined as $\Diamond_{\mathcal{I}} f = \top \,\mathcal{U}_{s,\mathcal{I}} f$ and $\Box_{\mathcal{I}} f = \neg(\Diamond_{\mathcal{I}} \neg f)$, respectively. The set $\mathcal{I}$ is a subset of $\mathbb{R}_{\geq 0} \times \mathbb{N}$ defining the hybrid time instances for which the properties stated by the operators should hold, as defined below.

An STL formula f being satisfied by a solution $(t, j) \mapsto \phi(t, j)$ at some time (t, j) is denoted by

$$(\phi, (t, j)) \vDash f. \tag{9}$$

Since an STL formula is a sentence consisting of atomic propositions and operators of STL, we can also consider an atomic proposition instead of a formula. Let p and q be atomic propositions. Given a solution ϕ to $\mathcal{H}$, $(t, j) \in \operatorname{dom}\phi$, and $\mathcal{I} \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$, the semantics of STL are defined as[3]

$$(\phi, (t, j)) \vDash \neg p \Leftrightarrow \neg((\phi, (t, j)) \vDash p), \tag{10a}$$

$$(\phi, (t, j)) \vDash p \vee q \Leftrightarrow (\phi, (t, j)) \vDash p \vee (\phi, (t, j)) \vDash q, \tag{10b}$$

$$(\phi, (t, j)) \vDash p \wedge q \Leftrightarrow (\phi, (t, j)) \vDash p \wedge (\phi, (t, j)) \vDash q, \tag{10c}$$

$$(\phi, (t, j)) \vDash \Diamond_{\mathcal{I}} p \Leftrightarrow \exists (t', j') \in \operatorname{dom}\phi \cap (\{(t, j)\} + \mathcal{I}) : (\phi, (t', j')) \vDash p, \tag{10d}$$

$$(\phi, (t, j)) \vDash \Box_{\mathcal{I}} p \Leftrightarrow (\phi, (t', j')) \vDash p \quad \forall (t', j') \in \operatorname{dom}\phi \cap (\{(t, j)\} + \mathcal{I}), \tag{10e}$$

$$\begin{aligned}(\phi, (t, j)) \vDash p\,\mathcal{U}_{s,\mathcal{I}} q \Leftrightarrow\; & \exists (t', j') \in \operatorname{dom}\phi \cap (\{(t, j)\} + \mathcal{I}) : (\phi, (t', j')) \vDash q \\ & \wedge \forall (t'', j'') \in \operatorname{dom}\phi \cap ([t, t'] \times \{j, j+1, \ldots, j'\}) \\ & \quad (\phi, (t'', j'')) \vDash p,\end{aligned} \tag{10f}$$

$$\begin{aligned}(\phi, (t, j)) \vDash p\,\mathcal{U}_{w,\mathcal{I}} q \Leftrightarrow\; & (\phi, (t', j')) \vDash p \\ & \forall (t', j') \in \operatorname{dom}\phi \cap (\{(t, j)\} + \mathcal{I}), \\ & \vee (\phi, (t, j)) \vDash p\,\mathcal{U}_{s,\mathcal{I}} q.\end{aligned} \tag{10g}$$

The same semantics of STL are used for formulas. For example, given an STL formula f, a solution ϕ satisfies $\Diamond_{\mathcal{I}} f$ at $(t, j) \in \operatorname{dom}\phi$ if the formula f holds at some time $(t', j') \in \operatorname{dom}\phi$ such that $(t', j') \in \{(t, j)\} + \mathcal{I}$.

Note that the STL syntax reduces to that of LTL when it is "untimed;" i.e., $\mathcal{I} = \mathbb{R}_{\geq 0} \times \mathbb{N}$. We introduce the following untimed strong until and weak until operators in LTL that will be used to certify STL formulas [15].

Definition 3.2 ($p\mathcal{U}_s q$)**.** *Given atomic propositions p and q, a solution ϕ to $\mathcal{H}$ satisfies the (untimed) formula $p\mathcal{U}_s q$ at $(t, j) \in \operatorname{dom}\phi$ if either $(\phi, (t, j)) \vDash q$; or*

- *there exists $(t', j') \in \operatorname{dom}\phi$ such that $t' + j' \geq t + j$, $(\phi, (t', j')) \vDash q$, and $(\phi, (t'', j'')) \vDash p$ for all $(t'', j'') \in \operatorname{dom}\phi \cap ([t, t'] \times \{j, j+1, \ldots, j'\})$.*

Definition 3.3 ($p\mathcal{U}_w q$)**.** *Given atomic propositions p and q, a solution ϕ to $\mathcal{H}$ satisfies the (untimed) formula $p\mathcal{U}_w q$ at $(t, j) \in \operatorname{dom}\phi$ if either*

- *$(\phi, (t', j')) \vDash p$ for all $(t', j') \in \operatorname{dom}\phi$ such that $t' + j' \geq t + j$; or*
- *ϕ satisfies $p\mathcal{U}_s q$ at (t, j).*

IV. Characterizations of STL Formulas $\Box_{\mathcal{I}} p$ and $\Diamond_{\mathcal{I}} p$ for Hybrid Dynamical Systems

In this section, we present characterizations of STL formulas $\Box_{\mathcal{I}} p$ and $\Diamond_{\mathcal{I}} p$ for hybrid dynamical systems.

[3]Given sets S_1 and S_2, $S_1 + S_2 := \{s_1 + s_2 \;:\; s_1 \in S_1, s_2 \in S_2\}$.

A. The STL always operator

For a hybrid system $\mathcal{H}=(C,F,D,G)$ as in (7), instead of using $\mathcal{H}_{f,\tau}$, we define a new hybrid system denoted $\mathcal{H}_\tau=(C_\tau,F_\tau,D_\tau,G_\tau)$, with state $\zeta=(x,\tau,k)\in\mathbb{R}^n\times\mathbb{R}_{\geq 0}\times\mathbb{N}$ and hybrid dynamics

$$\begin{aligned}\begin{bmatrix}\dot{x}\\ \dot{\tau}\\ \dot{k}\end{bmatrix}&\in F_\tau(x,\tau,k):=\begin{bmatrix}F(x)\\ 1\\ 0\end{bmatrix} \qquad \zeta\in C_\tau:=C\times\mathbb{R}_{\geq 0}\times\mathbb{N}\\ \begin{bmatrix}x^+\\ \tau^+\\ k^+\end{bmatrix}&\in G_\tau(x,\tau,k):=\begin{bmatrix}G(x)\\ \tau\\ k+1\end{bmatrix} \qquad \zeta\in D_\tau:=D\times\mathbb{R}_{\geq 0}\times\mathbb{N}.\end{aligned} \tag{11}$$

Note that for each solution[4] $\psi=(\varphi,\tau,k)$ to $\mathcal{H}_\tau$ with $\tau(0,0)=0$ and $k(0,0)=0$, the solution component φ is a solution to $\mathcal{H}$. To characterize STL formulas for $\mathcal{H}$, we introduce the set $\mathcal{I}\subset\mathbb{R}_{\geq 0}\times\mathbb{N}$ defined as[5]

$$\mathcal{I}:=[T_{\min},T_{\max}]\times\{J_{\min},J_{\min}+1,\ldots,J_{\max}\}, \tag{12}$$

where $T_{\min},J_{\min}\geq 0$, $T_{\max}\geq T_{\min}$, and $J_{\max}\geq J_{\min}$. Next, we establish conditions for the certification of $\Box_{\mathcal{I}}p$ and $\Diamond_{\mathcal{I}}p$ for $\mathcal{H}$. To this end, we extend P in (2) to

$$P:=\{x\in X:p(x)=1\}, \quad X:=\overline{C}\cup D\cup G(D) \tag{13}$$

where X collects the range of possible values of solutions.

The satisfaction of $\Box_{\mathcal{I}}p$ for each solution to $\mathcal{H}$ at $(t,j)=(0,0)$ is assured by guaranteeing the following properties for the solutions $\psi=(\varphi,\tau,k)$ to $\mathcal{H}_\tau$ starting from $X_o\times\{0\}\times\{0\}$, where $X_o\subset\mathbb{R}^n$ denotes the set of initial conditions:

- Each solution ψ to $\mathcal{H}_\tau$ stays in $X\times\{(\tau,k)\in\mathbb{R}_{\geq 0}\times\mathbb{N}:\tau+k<T_{\min}+J_{\min}\}$ until reaching $P\times\mathcal{I}$; and
- Once a solution ψ reaches $P\times\mathcal{I}$, ψ stays in $P\times\mathcal{I}$ until reaching $X\times\{(\tau,k)\in\mathbb{R}_{\geq 0}\times\mathbb{N}:\tau+k>T_{\max}+J_{\max}\}$.

The property in the first item holds for free by construction of $\mathcal{H}_\tau$, as long as there exists $(t,j)\in\operatorname{dom}\phi$ such that $t+j\geq T_{\max}+J_{\max}$. On the other hand, the property in the second item requires showing that the component φ of ψ remains in P over the hybrid time window defined by $\mathcal{I}$.

Now, we define atomic propositions p_a, p_b, and p_c as follows:

$$\begin{aligned}p_a(\tau,k)&:=\begin{cases}1 & \text{if } \tau+k<T_{\min}+J_{\min}\\ 0 & \text{otherwise,}\end{cases}\\ p_b(x,\tau,k)&:=\begin{cases}1 & \text{if } x\in P,(\tau,k)\in\mathcal{I}\\ 0 & \text{otherwise,}\end{cases}\\ p_c(\tau,k)&:=\begin{cases}1 & \text{if } \tau+k>T_{\max}+J_{\max}\\ 0 & \text{otherwise.}\end{cases}\end{aligned} \tag{14}$$

Next, we present a result establishing the satisfaction of STL always from properties of solutions to $\mathcal{H}_\tau$.

Theorem 4.1 (STL always operator)**.** *Given $\mathcal{H}=(C,F,D,G)$ as in* (7) *and an atomic proposition p, let P be given as in* (13)*. Given $\mathcal{I}$ as in* (12)*, let $\mathcal{H}_\tau$ be as in* (11) *and atomic propositions p_a, p_b, and p_c be as in* (14)*. Given a set $X_o\subset\mathbb{R}^n$, if the (untimed) formula $\tilde{f}=p_a\,\mathcal{U}_s(p_b\,\mathcal{U}_s p_c)$ is satisfied for each solution to $\mathcal{H}_\tau$ from $X_o\times\{0\}\times\{0\}$ at $(t,j)=(0,0)$, then the formula $f=\Box_{\mathcal{I}}p$ is satisfied for each solution to $\mathcal{H}$ from X_o at $(t,j)=(0,0)$.*

Proof. Suppose that $\tilde{f}=p_a\,\mathcal{U}_s(p_b\,\mathcal{U}_s p_c)$ is satisfied for each solution to $\mathcal{H}_\tau$ from $X_o\times\{0\}\times\{0\}$ at $(t,j)=(0,0)$. We show that each solution ϕ to $\mathcal{H}$ from X_o stays in P for all $(t,j)\in\mathcal{I}\cap\operatorname{dom}\phi$. Let $\psi=(\varphi,\tau,k)$ be a solution to $\mathcal{H}_\tau$ from $\varphi(0,0)\in X_o$, $\tau(0,0)=0$, and $k(0,0)=0$. The solution component φ is a solution (denoted ϕ) to $\mathcal{H}$ since the systems $\mathcal{H}$ and $\mathcal{H}_\tau$ implement the same dynamics for the state x. Furthermore, note that the components τ and k of ψ satisfy $\tau(t,j)=t$ and $k(t,j)=j$ for all $(t,j)\in\operatorname{dom}\phi$. Moreover, since, by assumption, $\tilde{f}=p_a\,\mathcal{U}_s(p_b\,\mathcal{U}_s p_c)$ is satisfied for each solution to $\mathcal{H}_\tau$ from $X_o\times\{0\}\times\{0\}$ at $(t,j)=(0,0)$, by definition of the $\mathcal{U}_s$ operator, the following hold:

- At times (t,j) at which (φ,τ,k) does not satisfy $p_b\,\mathcal{U}_s p_c$, (φ,τ,k) satisfies p_a, which from (14) implies that $\tau(t,j)+k(t,j)<T_{\min}+J_{\min}$;
- At times (t,j) at which (φ,τ,k) satisfies $p_b\,\mathcal{U}_s p_c$, (φ,τ,k) satisfies p_b until satisfying p_c; namely, $(\varphi(t,j),\tau(t,j),k(t,j))\in P\times\mathcal{I}$ at least until $\tau(t,j)+k(t,j)>T_{\max}+J_{\max}$.

Hence, we conclude that each solution ϕ to $\mathcal{H}$ with $\phi(0,0)=\varphi(0,0)\in X_o$ is such that $\phi(t,j)=\varphi(t,j)$ satisfies p for all $(t,j)\in\mathcal{I}\cap\operatorname{dom}\phi$, which implies that $f=\Box_{\mathcal{I}}p$ is satisfied for every solution ϕ to $\mathcal{H}$ at $(t,j)=(0,0)$. $\square$

B. The STL eventually operator

To certify $f=\Diamond_{\mathcal{I}}p$, as indicated above, we redefine atomic proposition p_b and p_c as follows:

$$\begin{aligned}p_b(\tau,k)&:=\begin{cases}1 & \text{if } (\tau,k)\in\mathcal{I}\\ 0 & \text{otherwise,}\end{cases}\\ p_c(x,\tau,k)&:=\begin{cases}1 & \text{if } x\in P,(\tau,k)\in\mathcal{I}\\ 0 & \text{otherwise.}\end{cases}\end{aligned} \tag{15}$$

Theorem 4.2 (STL eventually operator)**.** *Given $\mathcal{H}=(C,F,D,G)$ as in* (7) *and an atomic proposition p, let P be given as in* (13)*. Given $\mathcal{I}$ as in* (12)*, let $\mathcal{H}_\tau$ be as in* (11)*, and let the atomic proposition p_a be as in* (14) *and the atomic propositions p_b and p_c be as in* (15)*. Given a set $X_o\subset\mathbb{R}^n$, if the (untimed) formula $\tilde{f}=p_a\,\mathcal{U}_s(p_b\,\mathcal{U}_s p_c)$ is satisfied for each solution to $\mathcal{H}_\tau$ from $X_o\times\{0\}\times\{0\}$ at $(t,j)=(0,0)$ then the formula $f=\Diamond_{\mathcal{I}}p$ is satisfied for each solution to $\mathcal{H}$ from X_o at $(t,j)=(0,0)$.*

Proof. Suppose that $\tilde{f}=p_a\,\mathcal{U}_s(p_b\,\mathcal{U}_s p_c)$ is satisfied for a solution (φ,τ,k) to $\mathcal{H}_\tau$ at $(t,j)=(0,0)$ with $\varphi(0,0)\in X_o$, $\tau(0,0)=0$, and $k(0,0)=0$. We show that, for each solution ϕ to $\mathcal{H}$, there exists $(t,j)\in\mathcal{I}\cap\operatorname{dom}\phi$ such that $\phi(t,j)\in P$. Let (φ,τ,k) be a solution to $\mathcal{H}_\tau$ such that $\tau(0,0)=0$, $k(0,0)=0$, and $\varphi(t,j)=\phi(t,j)$ for all $(t,j)\in\operatorname{dom}\phi$. The solution component φ is a solution ϕ to $\mathcal{H}$ since the systems $\mathcal{H}$ and $\mathcal{H}_\tau$ implement the same dynamics for the

[4]As solutions to $\mathcal{H}$ in (7) are denoted by ϕ, we denote by ψ the solutions to $\mathcal{H}_\tau$ in (11).

[5]For compatibility with the STL literature, we define it as a compact set, but the unbounded set case can be treated similarly.

state x; and we note that $\tau(t,j) = t$ and $k(t,j) = j$ for all $(t,j) \in \operatorname{dom}\phi$. Since $\tilde{f} = p_a \mathcal{U}_s(p_b \mathcal{U}_s p_c)$ is satisfied for each solution (φ, τ, k) to $\mathcal{H}_\tau$ at $(t,j) = (0,0)$ with $\varphi(0,0) \in X_o$, $\tau(0,0) = 0$, and $k(0,0) = 0$, by definition of $\mathcal{U}_s$,

- At times (t,j) at which (φ, τ, k) does not satisfy $p_b \mathcal{U}_s p_c$, (φ, τ, k) satisfies p_a, which implies that $\tau(t,j) + k(t,j) < T_{\min} + J_{\min}$;
- At times (t,j) at which (φ, τ, k) satisfies $p_b \mathcal{U}_s p_c$, (φ, τ, k) satisfies p_b until satisfying p_c; namely, $(\varphi(t,j), \tau(t,j), k(t,j)) \in \mathbb{R}^n \times \mathcal{I}$ until, for some $(t',j') \in \operatorname{dom}(\varphi, \tau, k)$, $(\varphi(t',j'), \tau(t',j'), k(t',j')) \in P \times \mathcal{I}$.

Hence, we conclude that each solution ϕ to $\mathcal{H}$ such that $\phi(t,j) = \varphi(t,j)$ satisfies p for all $(t,j) \in \mathcal{I}$, which implies that $f = \Diamond_{\mathcal{I}} p$ is satisfied at $(t,j) = (0,0)$ for every solution ϕ to $\mathcal{H}$ with $\phi(0,0) \in X_o$. □

C. Satisfaction of the (untimed) formula $\tilde{f} = p_a \mathcal{U}_s(p_b \mathcal{U}_s p_c)$

With the sufficient conditions established in Theorem 4.1 and Theorem 4.2, we formulate sufficient conditions guaranteeing the (untimed) formula $\tilde{f} = p_a \mathcal{U}_s(p_b \mathcal{U}_s p_c)$. For simplicity, we consider the case when $X_o = X$. To this end, consider the hybrid system $\mathcal{H}_\tau$ as in (11) and let a closed set Q be given. Following [15], we build the auxiliary system $\mathcal{H}_w = (C_w, F_w, D_w, G_w)$ with state $\zeta = (x, \tau, k)$ and data given by

$$\begin{aligned} F_w(\zeta) &:= F_\tau(\zeta) & \forall \zeta \in C_w &:= C_\tau \setminus Q \\ G_w(\zeta) &:= \begin{cases} \zeta & \text{if } \zeta \in Q \\ G_\tau(\zeta) & \text{if } \zeta \in D_\tau \setminus Q \end{cases} & \forall \zeta \in D_w &:= D_\tau \cup Q. \end{aligned} \tag{16}$$

The intuition behind the construction of system $\mathcal{H}_w$ is to characterize the behavior of the system $\mathcal{H}_\tau$ outside the set Q. Indeed, the solutions to $\mathcal{H}_\tau$ are the solutions to $\mathcal{H}_w$ (and vice versa) up to when they reach (if they do) the set Q. Moreover, with a closed set $P \subset C_\tau \cup D_\tau$, we build another auxiliary system

$$\mathcal{H}_s = (C_s, F_s, D_s, G_s), \tag{17}$$

where $F_s := F_\tau$ with $C_s \subset \operatorname{dom} F_s$, $C_s := C_w \cap (P \cup Q) (= C_w \cap P)$, $G_s := G_w$ with $D_s \subset \operatorname{dom} G_s$, and $D_s := D_w \cap (P \cup Q)$. Note that $\mathcal{H}_s$ can be interpreted as the restriction of $\mathcal{H}_w$ in (16) to $P \cup Q$.

Theorem 4.3. *Consider $\mathcal{H}_\tau$ as in* (11) *with F outer semicontinuous and locally bounded with nonempty and convex values on C, and G having nonempty images on D. Given atomic propositions p_a, p_b, and p_c, let the sets P_a, P_b, and P_c be as in* (13) *while replacing p by p_a, p_b, and p_c, respectively, with x therein replaced by (x, τ, k), be such that P_a and $P_b \cup P_c$ are disjoint. The formula $\tilde{f} = p_a \mathcal{U}_s(p_b \mathcal{U}_s p_c)$ is satisfied for each solution to $\mathcal{H}_\tau$ starting from $\widetilde{X}_o := X \times \{0\} \times \{0\}$ at $(t,j) = (0,0)$ if*

1a) $P_a \cup P_b \cup P_c$ is conditionally invariant[6] *with respect to P_a for $\mathcal{H}_w$ with $Q = P_b \cup P_c$;*

1b) $P_b \cup P_c$ is eventually conditionally invariant[7] *with respect to $P_a \cup P_b \cup P_c$ for $\mathcal{H}_s$, or $P_b \cup P_c$ is finite time attractive*[8] *with respect to P_a for $\mathcal{H}_s$, both with $P = P_a$ and $Q = P_b \cup P_c$.*

and

2a) $P_b \cup P_c$ is conditionally invariant with respect to P_b for $\mathcal{H}_w$ with $Q = P_c$;

2b) P_c is eventually conditionally invariant with respect to $P_b \cup P_c$ for $\mathcal{H}_s$, or P_c is finite time attractive with respect to P_b for $\mathcal{H}_s$ with $P = P_b$ and $Q = P_c$.

Proof. We employ $\mathcal{H}_w$ and $\mathcal{H}_s$ defined in (16) and (17), respectively, and results in [15] to establish the claim. Note that item 1a) and item 1b) imply that $p_a \mathcal{U}_s(p_b \vee p_c)$ is satisfied for each solution to $\mathcal{H}_\tau$ starting from $\mathbb{R}^n \times \{0\} \times \{0\}$ at $(t,j) = (0,0)$. First, we apply results in [15] with P and Q therein given by $P = P_a$ and $Q = P_b \cup P_c$.

i) [15, Theorem 3.3] implies that $p_a \mathcal{U}_w(p_b \vee p_c)$ is satisfied for each solution to $\mathcal{H}_\tau$ starting from $X \times \{0\} \times \{0\}$ at $(t,j) = (0,0)$ if $P \cup Q$ is conditionally invariant with respect to $P \setminus Q$ for $\mathcal{H}_w$ in (16). The latter property holds by item 1a) since $P \cup Q = P_a \cup P_b \cup P_c$ and $P \setminus Q = P_a$ by construction of P and Q plus P_a and $P_b \cup P_c$ being disjoint.

ii) Next, we apply [15, Theorem 3.6] to show the following: (*) $p_a \mathcal{U}_s(p_b \vee p_c)$ is satisfied for each solution to $\mathcal{H}_\tau$ starting from $X \times \{0\} \times \{0\}$ at $(t,j) = (0,0)$. From item i) above, item 1) in [15, Theorem 3.6] holds. Item 2) requires the set Q to be eventually conditionally invariant with respect to $P \cup Q$ for $\mathcal{H}_s$. Since $Q = P_b \cup P_c$ and $P \cup Q = P_a \cup P_b \cup P_c$, item 2) in [15, Theorem 3.6] holds by virtue of the first part of item 1b). Instead, if the second part of item 1b) holds, (*) holds by an application of [15, Theorem 3.10].

Therefore, $p_a \mathcal{U}_s(p_b \vee p_c)$ is satisfied for each solution to $\mathcal{H}_\tau$ starting from $X \times \{0\} \times \{0\}$ at $(t,j) = (0,0)$.

To conclude the proof, we show that $p_b \mathcal{U}_s p_c$ is satisfied when items 2a) and 2b) hold using [15, Theorem 3.6]. Following the steps in items i) and ii) above, but with $P = P_b$ and $Q = P_c$ in [15, Theorem 3.6], item 2a) implies that item 1) in [15, Theorem 3.3] holds and, in turn, when item 2b) holds, [15, Theorem 3.6] implies that $p_b \mathcal{U}_s p_c$ is satisfied for each solution to $\mathcal{H}_\tau$ starting from $P_b \cup P_c$ at $(t,j) = (0,0)$. Hence, $\tilde{f}$ is satisfied since solutions from $\widetilde{X}_o$ result in $p_a \mathcal{U}_s(p_b \vee p_c)$ being true at a point in $P_b \cup P_c$. □

Remark 4.4. *Due to the characterization we provide, any sufficient condition that guarantees the satisfaction of the formula $\tilde{f} = p_a \mathcal{U}_s(p_b \mathcal{U}_s p_c)$ with the appropriate atomic propositions p_a, p_b, and p_c guarantees the satisfaction of the*

[6] Given sets $K \subset \mathbb{R}^n$ and $\mathcal{X}_o \subset K$, the set K is said to be conditionally invariant with respect to the set $\mathcal{X}_o$ for $\mathcal{H}$ if for each solution $\phi \in \mathcal{S}_{\mathcal{H}}(\mathcal{X}_o)$, $\phi(t,j) \in K$ for all $(t,j) \in \operatorname{dom}\phi$.

[7] Given sets $\mathcal{O} \subset \overline{C} \cup D$ and $\mathcal{A} \subset \mathbb{R}^n$, the set $\mathcal{A}$ is said to be eventually conditionally invariant with respect to $\mathcal{O}$ for $\mathcal{H}$ if for each maximal solution $\phi \in \mathcal{S}_{\mathcal{H}}(\mathcal{O})$, there exists $(t,j) \in \operatorname{dom}\phi$ such that $\phi(t',j') \in \mathcal{A}$ for all $t' + j' \geq t + j$.

[8] Given sets $\mathcal{O} \subset \overline{C} \cup D$ and $\mathcal{A} \subset \mathbb{R}^n$ such that $\mathcal{A}$ is closed, the set $\mathcal{A}$ is said to be finite-time attractive with respect to $\mathcal{O}$ for $\mathcal{H}$ if for each solution $\phi \in \mathcal{S}_{\mathcal{H}}(\mathcal{O})$, $\mathcal{T}_{\mathcal{A}}(\phi) < \infty$, where $\mathcal{T}_{\mathcal{A}} : \mathcal{S}_{\mathcal{H}}(\mathcal{O}) \mapsto \mathbb{R}_{\geq 0}$ is the settling-time function providing (when finite) the first hybrid time at which the solution ϕ reaches the set $\mathcal{A}$.

formulas $\Box_{\mathcal{I}} p$ and $\Diamond_{\mathcal{I}} p$. For example, the results in terms of barrier functions or Lyapunov-like functions conditions in [15, Section 6] can be applied to formulate sufficient conditions for the satisfaction of the formula $\tilde{f} = p_a \mathcal{U}_s (p_b \mathcal{U}_s p_c)$.

Example 4.5 (Robotic Manipulation (revisited))**.** *Consider the robotic manipulation problem in Example 2.1 where the continuous dynamics of a controlled end effector are given by* (5)*. When the velocity of the end effector is large, contacts result in impacts between the end effector and the surface. Modeling the impacts using the jump map G and the jump set D in* (7)*, a controlled end effector is given by the hybrid system with state $x = (x_1, x_2) \in \mathbb{R}^2$ and dynamics*

$$\mathcal{H} \quad \begin{cases} \dot{x} \in \begin{bmatrix} x_2 \\ F_2(x) \end{bmatrix} =: F(x) & x \in C \\ x^+ = \begin{bmatrix} x_1 \\ -e_R x_2 \end{bmatrix} =: G(x) & x \in D \end{cases} \tag{18}$$

where $C := \{x \in L_V(c) : x_1 \leq 0\} \cup \{x \in \mathbb{R}^2 : x_1 \geq 0, x_2 \leq \bar{x}_2\}$, $D := \{x \in L_V(c) : x_1 \geq 0, x_2 \geq \bar{x}_2\}$, $e_R \in [0,1]$ is the restitution coefficient, $\bar{x}_2$ denotes the lower velocity threshold at which contacts are treated as impacts, and $L_V(c) := \{x \in \mathbb{R}^2 : V(x) \leq c\}$ with $x \mapsto V(x) := \frac{k_p}{2} x_1^2 + \frac{1}{2} x_2^2$ and $c > 0$. It follows from [16, Proposition 1] that $X = C \cup D \cup G(D)$, which is compact, is such that every solution to (18) *from X stays in X; i.e., X is forward invariant for* (18)*. Note that maximal solutions to this system exhibit jumps when $\bar{x}_2$ is small enough, namely, when D is nonempty; see the numerical results in [16, Section 5.1].*

Next, consider $f = \Diamond_{\mathcal{I}} p$ with p as in (6) *with $\mathcal{I}$ in* (12) *with $0 = T_{\min} < T_{\max}$ and $J_{\min} = J_{\max} = 0$, enforcing that p should hold over the first interval of flow within the ordinary time window $[0, T_{\max}]$, with $T_{\max}$ to be defined. To certify this formula, we apply Theorem 4.2 and Theorem 4.3, for which we consider $\mathcal{H}_\tau$ in* (11)*. Note that P in* (13) *is given by $\{x \in X : x_1 \leq 0, x_2 \geq 0\}$. Using p_a in* (14)*, and p_b and p_c in* (15)*, the sets P_a, P_b, and P_c are given by $P_a = X \times [0, T_{\min}) \times \{0\}$, $P_b = X \times [T_{\min}, T_{\max}] \times \{0\}$, and $P_c = P \times [T_{\min}, T_{\max}] \times \{0\}$, respectively. Item 1a) in Theorem 4.3 holds since the associated system $\mathcal{H}_w$ in* (16) *is such that all of its maximal solutions end in Q in finite time; hence, $P_a \cup P_b \cup P_c$ is conditionally invariant with respect to P_a for $\mathcal{H}_w$ Similarly, item 1b) holds since the maximal solutions to $\mathcal{H}_s$ also terminate in Q, leading finite time attractivity of $P_b \cup P_c$ with respect to P_a. To satisfy items 2a) and 2b), given $c > 0$ and defining the compact set X as above, $T_{\max}$ is chosen to be larger than the time required for the position component of any solution to* (6) *to reach zero with nonnegative velocity. With this choice, and since the compact set X is forward invariant, maximal solutions to $\mathcal{H}_w$ from P_b reach P_c; hence, item 2a) holds. Showing that item 2b) holds follows similarly. Hence, since (C, F, D, G) satisfy the assumptions in Theorem 4.3, through an application of Theorem 4.2, the formula $f = \Diamond_{\mathcal{I}} p$ is satisfied for each solution to $\mathcal{H}$ in* (18) *from $X_o = X$ at $(t, j) = (0, 0)$.*

V. Conclusion

Semantics and characterization for the certification of $\Box_{\mathcal{I}} p$ and $\Diamond_{\mathcal{I}} p$ are presented by exploiting invariance properties for dynamical systems. Equivalence relationships are established between the satisfaction of LTL formulas having until operators and the satisfaction of STL formulas with always and until operators. As a result, sufficient conditions guaranteeing the satisfaction of STL formulas are proposed by guaranteeing the satisfaction of LTL formulas involving until operators. Future research pertains to guaranteeing other STL operators and associated sufficient conditions, by exploiting the ideas in Remark 4.4 and in [14, Section 6], and to assuring robustness of STL specifications.

References

[1] P. Tabuada and G. J. Pappas, "Linear time logic control of discrete-time linear systems," IEEE Transactions on Automatic Control, vol. 51, no. 12, pp. 1862–1877, 2006.

[2] Y. Kwon and G. Agha, "Ltlc: Linear temporal logic for control," in Hybrid Systems: Computation and Control: 11th International Workshop, HSCC 2008, St. Louis, MO, USA, April 22-24, 2008. Proceedings 11. Springer, 2008, pp. 316–329.

[3] O. Maler and D. Nickovic, "Monitoring temporal properties of continuous signals," in Formal Techniques, Modelling and Analysis of Timed and Fault-Tolerant Systems. Springer, 2004, pp. 152–166.

[4] R. Alur, T. Feder, and T. A. Henzinger, "The benefits of relaxing punctuality," Journal of the ACM (JACM), vol. 43, no. 1, pp. 116–146, 1996.

[5] H. Roehm, J. Oehlerking, T. Heinz, and M. Althoff, "Stl model checking of continuous and hybrid systems," in Automated Technology for Verification and Analysis: 14th International Symposium, ATVA 2016, Chiba, Japan, October 17-20, 2016, Proceedings 14. Springer, 2016, pp. 412–427.

[6] G. E. Fainekos and G. J. Pappas, "Robustness of temporal logic specifications for continuous-time signals," Theoretical Computer Science, vol. 410, no. 42, pp. 4262–4291, 2009.

[7] A. Donzé and O. Maler, "Robust satisfaction of temporal logic over real-valued signals," in International Conference on Formal Modeling and Analysis of Timed Systems. Springer, 2010, pp. 92–106.

[8] G. Yu, J. Lee, and K. Bae, "Stlmc: Robust stl model checking of hybrid systems using smt," in Computer Aided Verification: 34th International Conference, CAV 2022, Haifa, Israel, August 7–10, 2022, Proceedings, Part I. Springer, 2022, pp. 524–537.

[9] K. Bae and J. Lee, "Bounded model checking of signal temporal logic properties using syntactic separation," Proceedings of the ACM on Programming Languages, vol. 3, no. POPL, pp. 1–30, 2019.

[10] A. Donzé, T. Ferrere, and O. Maler, "Efficient robust monitoring for stl," in International Conference on Computer Aided Verification. Springer, 2013, pp. 264–279.

[11] V. Raman, A. Donzé, D. Sadigh, R. M. Murray, and S. A. Seshia, "Reactive synthesis from signal temporal logic specifications," in Proceedings of the 18th International Conference on Hybrid Systems: Computation and Control. ACM, 2015, pp. 239–248.

[12] L. Lindemann and D. V. Dimarogonas, "Robust control for signal temporal logic specifications using discrete average space robustness," Automatica, vol. 101, pp. 377–387, 2019.

[13] R. Goebel, R. G. Sanfelice, and A. R. Teel, Hybrid Dynamical Systems: modeling, stability, and robustness. Princeton University Press, 2012.

[14] H. Han and R. G. Sanfelice, "Linear temporal logic for hybrid dynamical systems: Characterizations and sufficient conditions," Nonlinear Analysis: Hybrid Systems, vol. 36, p. 100865, 2020.

[15] H. Han, M. Maghenem, and R. G. Sanfelice, "Certifying the LTL formula p until q in hybrid systems," To appear in IEEE Transactions on Automatic Control, 2023. [Online]. Available: https://arxiv.org/pdf/2106.06455.pdf

[16] R. Naldi and R. G. Sanfelice, "Passivity-based control for hybrid systems with applications to mechanical systems exhibiting impacts," Automatica, vol. 49, no. 5, p. 1104–1116, May 2013. [Online]. Available: https://hybrid.soe.ucsc.edu/files/preprints/69.pdf