Passivity-based Controllers for a Class of Hybrid Systems with Applications to Mechanical Systems Interacting with their Environment

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***Abstract*— Motivated by applications of systems interacting with their environments, we study the design of passivity-based controllers for a class of hybrid systems. Classical and hybrid-specific notions of passivity along with detectability and solution conditions are linked to asymptotic stability. These results are used to design passivity-based controllers following classical passivity theory. An application, pertaining to a point mass physically interacting with the environment, illustrates the definitions and the results obtained throughout this work.**

I. Introduction

Dissipativity and its special case, passivity, provide a useful physical interpretation to stability and stabilizability problems as they establish a relationship between the energy injected in and dissipated by a system. Their application in both the analysis and the design of control systems has been the subject of several textbooks [1], [2], [3], [4] and seminal papers [5], [6], [7], [8], [9]. Moreover, the passivity-based control design technique has been shown to be particularly useful in designing controllers that can be well understood from an energetic perspective. The problem of stabilizing a system to a given equilibrium point, in particular, is addressed by designing a feedback controller such that the overall energy function has the desired form and minimum, and by selecting the input so that the energy of the system is dissipated (see, e.g., [7]).

Dissipativity and passivity have been recently considered for several types of hybrid systems. Passivity of switching systems was investigated in [10]. Motivated by haptic and teleoperation applications, a notion of passivity for systems in which the controller switches between different operative modes was proposed in [11]. Results about dissipativity of switching systems appeared also in [12], where multiple storage functions were considered. Passivity and passivity-based control for systems undertaking impacts and unilateral constraints have been investigated in [13]. The results are applied to mechanical systems including robotic manipulators with rigid or flexible joints. In [14], passivity-based control techniques are employed to regulate walking for a class of bipedal robots (see also [15]). Impact Poincaré maps are considered as a tool to investigate stability of the periodic orbits characterizing the desired walking behavior. In [16], the authors consider dissipativity theory for a class of impulsive dynamical systems. In particular, the framework in [16] considers different inputs and outputs maps for respectively the continuous-time evolution and the instantaneous changes, and results linking observability to asymptotic stability for the design of feedback controllers are presented. More recently, a general notion of dissipativity for a class of hybrid systems was linked to detectability and used to establish asymptotic stability for large-scale interconnections of hybrid systems in [17].

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Building from the ideas in [16] and [17], and driven by an application of a mechanical system interacting with the environment, this paper studies the design of passivity-based controllers for a class of hybrid systems. In particular, we study the case of hybrid systems in which the energy dissipation may only happen along either the continuous or the discrete dynamics. For such systems, a weaker notion of passivity, encompassing the definition given in [16], is introduced and shown how it can be linked to asymptotic stability. For this purpose, a notion of detectability is employed. The result is then applied to an application that consists of a mechanical system capturing the dynamics of a simple robotic manipulator (see also [18], [19]) that is required to interact physically with the environment through the effect of a control input affecting the continuous dynamics.

The remainder of the paper is organized as follows. In Section II, the application is presented. Section III presents the general definition of passivity and the conditions to link this property to asymptotic stability. In Section IV, a passivity-based control result is given and then applied to the special passivity case of the application. Numerical results are then presented in Section V.

II. Motivational Application

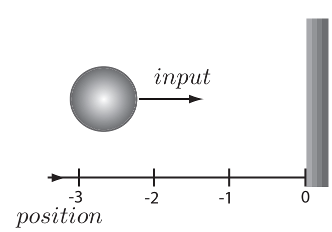

Fig. 1. Motivational application: a point mass interacting with the environment.

We consider the mechanical system depicted in Figure 1, which consists of a point mass driven by a controlled force. The mass is constrained to move horizontally and, during its motion, it may come into contact with a surface located at the origin of the line of motion. The position and the velocity of the mass have been denoted with x_1 and x_2, respectively. In order to model collisions between the ball and the surface, inspired also by [20], we consider a discontinuous contact model that depends on the velocity of the system at impacts. More specifically, when the impact velocity is lower than a certain threshold, denoted as $\bar{x}_2 > 0$, the mass is subject to a contact force that depends on the viscoelastic properties of the contact material. Assuming unitary mass for sake of simplicity, the system is described by the following equations:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = v_c - f_c(x), \tag{1}$$

where $v_c \in \mathbb{R}$ denotes the input force, $f_c(x)$ the contact force

$$f_c(x) = \begin{cases} k_c x_1 + b_c x_2 & \text{if } x_1 > 0 \\ 0 & \text{if } x_1 \leq 0 \end{cases}$$

in which $k_c > 0$ and $b_c > 0$ are, respectively, the elastic and damping coefficients of the compliant contact model.

On the other hand, when a collision with the surface occurs with a velocity of the mass greater or equal than $\bar{x}_2$, the impact is assumed to be impulsive and, accordingly, the rigid body instantaneously rebounds or jumps. The contact condition can be modeled as

$$x_1 \geq 0 \text{ and } x_2 \geq \bar{x}_2 \tag{2}$$

in which $x_1 = 0$ denotes the position of the vertical surface, while the new value of the state variables after the impact, denoted in the following with the superscript $+$, can be described by the reset law $x_1^+ = x_1$, $x_2^+ = -\varrho x_2$, where $\varrho \in [0, 1]$ represents the restitution coefficient.

Suppose that the control goal is to stabilize this simple mechanical system to a fixed position in contact with the vertical surface, say, the origin. Consider the quadratic function $V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$ and note that the following holds:

1) For each x such that (2) holds, since $x_1 = 0$ and $\varrho \in [0, 1]$,

$$V(x^+) = \frac{1}{2}x_1^2 + \frac{1}{2}\varrho^2 x_2^2 \leq \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 = V(x).$$

2) For each x not in (2), if $x_1 \leq 0$

$$\left\langle \nabla V(x), \begin{bmatrix} x_2 \\ v_c - f_c(x) \end{bmatrix} \right\rangle = x_2(x_1 + v_c)$$

and if $x_1 > 0$

$$\left\langle \nabla V(x), \begin{bmatrix} x_2 \\ v_c - f_c(x) \end{bmatrix} \right\rangle = x_2((1 - k_c)x_1 + v_c - b_c x_2)$$

Picking $v_c = -x_1 + w_c$ for $x_1 \leq 0$ and $v_c = -(1 - k_c)x_1 + b_c x_2 + w_c$ for $x_1 < 0$, where w_c is a new input, makes the right-hand side of the expressions in item 2) above to be equal to $x_2 w_c$. The resulting expressions imply that the variation of V during flows is no larger that the product $x_2 w_c$, which can be interpreted as a passivity property of the system with input w_c and output $y_c := x_2$. However, a similar passivity property does not seem to hold at jumps for this storage function. This motivates to investigate passivity-based control design methods for hybrid systems that are applicable when passivity holds only during one regime only.

III. General Definitions and Results

A. Passivity Notions

We consider hybrid systems $\mathcal{H}$ as in [21] given by[1]

$$\mathcal{H} \quad \begin{cases} \dot{x} \in F(x, v_c) & (x, v_c) \in C \\ x^+ \in G(x, v_d) & (x, v_d) \in D \\ y = h(x, v) \end{cases} \tag{3}$$

with state $x \in \mathbb{R}^n$, input $v = \left[v_c^\top, v_d^\top\right]^\top \in \mathbb{R}^m$ in which $v_c \in \mathbb{R}^{m_c}$ and $v_d \in \mathbb{R}^{m_d}$ are respectively the inputs acting on the flows and jumps, and output $y \in \mathbb{R}^p$. The sets $C \subset \mathbb{R}^n \times \mathbb{R}^{m_c}$ and $D \subset \mathbb{R}^n \times \mathbb{R}^{m_d}$ define the flow and jump sets, respectively; the set-valued mappings $F : \mathbb{R}^n \times \mathbb{R}^{m_c} \rightrightarrows \mathbb{R}^n$ and $G : \mathbb{R}^n \times \mathbb{R}^{m_d} \rightrightarrows \mathbb{R}^n$ define the flow map and jump map, respectively; finally the function $h : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ defines the output. Since only some components of the output y might be involved in the changes of energy during flows and jumps, we define $y_c = h_c(x, v_c) \in \mathbb{R}^{m_c}$ and $y_d = h_d(x, v_d) \in \mathbb{R}^{m_d}$, which corresponds to the case when the size of inputs v_c and v_d coincide with the size of the outputs y_c and y_d, respectively (property that in [4] is called *duality* of the output and input space).

For this class of hybrid systems we consider the following concept of passivity. Below, h_c, h_d and a compact set $\mathcal{A} \subset \mathbb{R}^n$ satisfy $h_c(\mathcal{A}, 0) = h_d(\mathcal{A}, 0) = 0$.

Definition 1: A hybrid system $\mathcal{H}$ for which there exists a function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$

- continuous on $\mathbb{R}^n$;
- continuously differentiable on a neighborhood of $\overline{C}$;
- satisfying for some functions $\omega_c : \mathbb{R}^{m_c} \times \mathbb{R}^n \to \mathbb{R}$ and $\omega_d : \mathbb{R}^{m_c} \times \mathbb{R}^n \to \mathbb{R}$

$$\langle \nabla V(x), \xi \rangle \leq \omega_c(v_c, x) \quad \forall (x, v_c) \in C, \xi \in F(x, v_c) \tag{4}$$
$$V(\xi) - V(x) \leq \omega_d(v_d, x) \quad \forall (x, v_d) \in D, \xi \in G(x, v_d) \tag{5}$$

called a *storage function*, is said to be

- *passive with respect to a compact set* $\mathcal{A}$ if

$$(v_c, x) \mapsto \omega_c(v_c, x) = v_c^\top y_c \tag{6}$$
$$(v_d, x) \mapsto \omega_d(v_d, x) = v_d^\top y_d. \tag{7}$$

  It is then called flow-passive (respectively, jump-passive) if it is passive with $\omega_d \equiv 0$ (respectively, $\omega_c \equiv 0$).
- *strictly passive with respect to a compact set* $\mathcal{A}$ if

$$(v_c, x) \mapsto \omega_c(v_c, x) = v_c^\top y_c - \rho_c(x)$$
$$(v_d, x) \mapsto \omega_d(v_d, x) = v_d^\top y_d - \rho_d(x),$$

[1] At times, for simplicity in the notation, we will drop the dependency on v on the data (C, F, D, G, h) and write, for example, $F(x)$ instead of $F(x, v)$ and $x \in C$ instead of $(x, v) \in C$.

where $\rho_c, \rho_d : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ are positive definite with respect to $\mathcal{A}$. It is then called flow-strictly passive (respectively, jump-strictly passive) if it is strictly passive with $\omega_d \equiv 0$ (respectively, $\omega_c \equiv 0$).

- *output strictly passive with respect to* $\mathcal{A}$ if

$$\begin{array}{rcl} (v_c, x) \mapsto \omega_c(v_c, x) & = & v_c^\top y_c - y_c^\top \rho_c(y_c) \\ (v_d, x) \mapsto \omega_d(v_d, x) & = & v_d^\top y_d - y_d^\top \rho_d(y_d), \end{array}$$

where $\rho_c : \mathbb{R}^{m_c} \to \mathbb{R}^{m_c}$, $\rho_d : \mathbb{R}^{m_d} \to \mathbb{R}^{m_d}$ are functions such that $y_c^\top \rho_c(y_c) > 0$ for all y with $y_c \neq 0$ and such that $y_d^\top \rho_d(y_d) > 0$ for all y with $y_d \neq 0$. It is then called flow-output strictly passive (respectively, jump-output strictly passive) if it is output strictly passive with $\omega_d \equiv 0$ (respectively, $\omega_c \equiv 0$).

The definitions of passivity above include the ones typically defined for the continuous and discrete-time settings as well as special cases when passivity holds only for the flow or jump equation. These special cases, denoted respectively as flow-passivity and jump-passivity, are motivated also by the application introduced in Section II, in which energy dissipation happens along flows, but not necessarily along jumps. It will be shown in Section III-C that such notion of passivity can be linked to asymptotic stability under weaker conditions than when using the standard notions. Passivity-based control techniques for such special cases will also be provided in Section IV.

1) Application revisited: Consider the mechanical system introduced in Section II. By considering the Filippov regularization of the discontinuous contact force $f_c(x)$ given by

$$f_c^r(x) = \begin{cases} k_c x_1 + b_c x_2 & \text{if } x_1 > 0 \\ \overline{\text{con}}\,\{0,\, b_c x_2\} & \text{if } x_1 = 0 \\ 0 & \text{if } x_1 < 0\,, \end{cases} \tag{8}$$

the mechanical system of interest can then be described by means of the following (regularized) hybrid system

$$\mathcal{H}_S \quad \begin{cases} \dot{x} \in F(x, v_c) := \begin{bmatrix} x_2 \\ v_c - f_c^r(x) \end{bmatrix} & x \in C \\ x^+ = G(x) := \begin{bmatrix} x_1 \\ -\varrho x_2 \end{bmatrix} & x \in D \end{cases} \tag{9}$$

with state $x = [x_1, x_2]^\top \in \mathbb{R}^2$, input $v_c \in \mathbb{R}$, and sets C and D given by

$$\begin{array}{rcl} C & := & \{x \in \mathbb{R}^2 : x_1 \leq 0\} \cup \{x \in \mathbb{R}^2 : x_1 \geq 0, x_2 \leq \bar{x}_2\} \\ D & := & \{x \in \mathbb{R}^2 : x_1 \geq 0,\, x_2 \geq \bar{x}_2\}\,. \end{array} \tag{10}$$

In the following we show how the control input v_c can be designed to obtain a new hybrid system, denoted as $\mathcal{H}_{S^1}$, which, by choosing as output $y_c = h_c(x) := x_2$, is *flow passive* with respect to the compact set $\mathcal{A} = (x_1^\star, 0)$, where $x_1^\star \geq 0$ denotes the desired set-point position for the mass. The choice $x_1^\star \geq 0$ requires the mass to maintain a contact with the vertical surface. Inspired by the *energy shaping* approach, see among others [7], which consists in assigning a desired potential energy to the closed-loop mechanical system, let the control input v_c in (9) be given by

$$v_c = v_c^\star(x_1, w_c) := \begin{cases} k_c x_1 - k_P(x_1 - x_1^\star) + w_c & \text{if } x_1 > 0 \\ -k_P(x_1 - x_1^\star) + w_c & \text{if } x_1 \leq 0 \end{cases} \tag{11}$$

in which $k_P > 0$ and $w_c \in \mathbb{R}$ is a new input. Accordingly, the resulting hybrid system is then given by

$$\mathcal{H}_{S^1} \quad \begin{cases} \dot{x} \in F_{S^1}(x, w_c) := \\ \qquad \begin{bmatrix} x_2 \\ v_c^\star(x_1, w_c) - f_c^r(x) \end{bmatrix} & x \in C \\ x^+ = G(x) \quad x \in D. \end{cases} \tag{12}$$

By considering the storage function

$$V(x) = \frac{1}{2} k_P (x_1 - x_1^\star)^2 + \frac{1}{2} x_2^2, \tag{13}$$

along flows we obtain (see [22])

$$\langle \nabla V(x), \eta \rangle \leq w_c y_c \qquad \forall \eta \in F_{S^1}(x, w_c)\,.$$

Along jumps we have $V(G(x)) - V(x) \leq -\frac{1}{2}(1-\varrho^2) y_c^2 \leq 0$ for all $x \in D$. The two properties above show that system (12) is flow-passive with respect to the compact set $\mathcal{A}$ with output y_c, input w_c, and function $\omega_c(w_c, x) := w_c y_c$.

Finally, the new input w_c in (12) can be designed to induce flow-output strict passivity. In particular, let the control input w_c in (11) be chosen as

$$w_c = -k_1 x_2 + \widetilde{w}_c \tag{14}$$

in which $k_1 > 0$ is the damping injection gain and $\widetilde{w}_c \in \mathbb{R}$ is a new control input. By considering the same storage function (13), with the choice (14) along flows it now holds

$$\langle \nabla V(x), \xi \rangle \leq \widetilde{w}_c y_c - k_1 y_c^2 \qquad \forall \xi \in F_{S^1}(x, w_c)\,.$$

Since we have that $V(G(x)) - V(x) \leq 0$, as shown above, system (12) with w_c given by (14) is flow-output strictly passive with respect to the compact set $\mathcal{A} = (x_1^\star, 0)$ with output $y_c = x_2$, input $\widetilde{w}_c$, and functions $\omega_c(\widetilde{w}_c, x) := \widetilde{w}_c y_c$ and $\rho_c(y_c) := k_1 y_c$.

B. Stability and Detectability Notions

In this work, for a hybrid system $\mathcal{H}$, we consider the notion of solution given in [23]. Moreover, we consider the following stability definitions for hybrid systems when their input is set to zero.

Definition 2: A compact set $\mathcal{A} \subset \mathbb{R}^n$ is said to be

- *0-input stable* if for each $\varepsilon > 0$ there exists $\delta > 0$ such that each maximal solution pair $(\phi, 0)$ to $\mathcal{H}$ and $\phi(0,0) = \xi$, $|\xi|_{\mathcal{A}} \leq \delta$, satisfies $|\phi(t,j)|_{\mathcal{A}} \leq \varepsilon$ for all $(t,j) \in \operatorname{dom}\phi$;
- *0-input pre-attractive* if there exists $\mu > 0$ such that every maximal solution pair $(\phi, 0)$ to $\mathcal{H}$ and $\phi(0,0) = \xi$, $|\xi|_{\mathcal{A}} \leq \mu$, is bounded and if it is complete satisfies

$$\lim_{(t,j) \in \operatorname{dom}\phi,\, t+j \to \infty} |\phi(t,j)|_{\mathcal{A}} = 0;$$

- *0-input pre-asymptotically stable* if it is 0-input stable and 0-input pre-attractive.

When every maximal solution is complete, the prefix "pre" can be removed. Asymptotic stability is said to be global when the attractivity property holds in $\overline{C} \cup D$.

We define a general detectability property for hybrid systems $\mathcal{H}$ with inputs set to zero. In the next section, this notion will permit linking passivity with stability.

Definition 3 (see Definition 6.2 in [24]): Given sets $\mathcal{A}$ and $K \subset \mathbb{R}^n$, the distance to $\mathcal{A}$ is 0-input detectable relative to K for $\mathcal{H}$ if every complete solution pair $(\phi, 0)$ to $\mathcal{H}$ such that

$$\phi(t,j) \in K \quad \forall (t,j) \in \operatorname{dom}\phi \Rightarrow \lim_{t+j\to\infty,\ (t,j)\in\operatorname{dom}\phi} |\phi(t,j)|_{\mathcal{A}} = 0. \tag{15}$$

If $\mathcal{H}$ does not have inputs, the distance to $\mathcal{A}$ is detectable relative to K for $\mathcal{H}$ if every complete solution ϕ to $\mathcal{H}$ satisfies (15).

When K is given by the set of points x such that $h(x,0) = 0$, the condition $\phi(t,j) \in K$ for all $(t,j) \in \operatorname{dom}\phi$ is equivalent to holding the output to zero. In such a case, Definition 3 reduces to the classical notion of detectability.

C. Basic Properties

We relate different forms of passivity to asymptotic stability with zero input, that is, for the hybrid system $\mathcal{H}$ with $v = 0$ given by

$$\mathcal{H}_0 \qquad \begin{cases} \dot{x} & \in & F(x,0) & (x,0) \in C \\ x^+ & \in & G(x,0) & (x,0) \in D \\ y & = & h(x,0). \end{cases} \tag{16}$$

Below, given a set $S \subset \mathbb{R}^n \times \mathbb{R}^m$, let

$$\Pi_0(S) := \{x \in \mathbb{R}^n \ : \ (x,0) \in S\}.$$

Also, we say that a set-valued mapping $\phi : S \rightrightarrows \mathbb{R}^n$ with $S \subset \mathbb{R}^n \times \mathbb{R}^m$ is outer semicontinuous relative to S if for any $z \in S$ and any sequence $\{z_i\}_{i=1}^{\infty}$ with $z_i \in S$, $\lim_{i\to\infty} z_i = z$, and any sequence $\{w_i\}_{i=1}^{\infty}$ with $w_i \in \phi(z_i)$ and $\lim_{i\to\infty} w_i = w$ we have $w \in \phi(z)$.

For the next proposition to hold, the data of $\mathcal{H}_0$ has to satisfy the following properties:

- (A1) The sets $\Pi_0(C)$ and $\Pi_0(D)$ are closed in $\mathbb{R}^n$.
- (A2) The set-valued mapping $(x,0) \mapsto F(x,0)$ is outer semicontinuous relative to $\mathbb{R}^n \times \{0\}$ and locally bounded, and for all $x \in \Pi_0(C)$, $F(x,0)$ is nonempty and convex.
- (A3) The set-valued mapping $(x,0) \mapsto G(x,0)$ is outer semicontinuous relative to $\mathbb{R}^n \times \{0\}$ and locally bounded, and for all $x \in \Pi_0(D)$, $G(x,0)$ is nonempty.

Observe that property (A1) simply requires that the set C and D are closed for the case in which $v = 0$.

Proposition 1: Given a compact set $\mathcal{A} \subset \mathbb{R}^n$, if the hybrid system $\mathcal{H}$ satisfying (A1)-(A3) is

1) passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$ then $\mathcal{A}$ is 0-input stable for $\mathcal{H}$.
2) output strict passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$ and the distance to $\mathcal{A}$ is detectable relative to
$$\begin{array}{l}\left\{x \in \Pi_0(C) \ : \ h_c(x,0)^\top \rho_c(h_c(x,0)) = 0\ \right\} \cup \\ \left\{x \in \Pi_0(D) \ : \ h_d(x,0)^\top \rho_d(h_d(x,0)) = 0\ \right\}\end{array} \tag{17}$$
for $\mathcal{H}_0$ then $\mathcal{A}$ is 0-input pre-asymptotically stable for $\mathcal{H}$.
3) strictly passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$ then $\mathcal{A}$ is 0-input pre-asymptotically stable for $\mathcal{H}$.

For the proof of the above proposition the reader is referred to [22].

Remark. The 0-input stability property of $\mathcal{A}$ in items 1 and 2 of Proposition 1 can be established without insisting on conditions (A1)-(A3). The attractivity property in item 2 requires these conditions due to the use of an invariance principle from [24]. Conditions (A1)-(A3) guarantee required structural properties of the solution set to $\mathcal{H}_0$, in particular, sequential compactness. The second item of Proposition 1 can also be asserted from [17, Theorem 2] (its proof does not use an invariance principle) when specializing the general dissipativity concept therein to the passivity case. The purpose of Proposition 1 is to enable the special cases that are considered in Proposition 2 below. ◁

The results given in Proposition 1 can also be applied to the special cases of flow and jump passivity given in Definition 1. However, for these latter cases, less conservative conditions can be obtained as shown in the following result whose proof is available in [22].

Proposition 2: Given a compact set $\mathcal{A} \subset \mathbb{R}^n$, if the hybrid system $\mathcal{H}$ satisfying (A1)-(A3) is

1) flow-passive or jump-passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$ then $\mathcal{A}$ is 0-input stable for $\mathcal{H}$.
2) flow-output strictly passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$ and
   - 2.a) the distance to $\mathcal{A}$ is detectable relative to
   $$\left\{x \in \Pi_0(C) \ : \ h_c(x,0)^\top \rho_c(h_c(x,0)) = 0\ \right\} \tag{18}$$
   for $\mathcal{H}_0$,
   - 2.b) every complete solution ϕ to $\mathcal{H}_0$ is such that for some $\delta > 0$ and some $J \in \mathbb{N}$ we have $t_{j+1} - t_j \geq \delta$ for all $j \geq J$,

   then $\mathcal{A}$ is 0-input pre-asymptotically stable for $\mathcal{H}$.
3) jump-output strictly passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$ and,
   - 3.a) the distance to $\mathcal{A}$ is detectable relative to
   $$\left\{x \in \Pi_0(D) \ : \ h_d(x,0)^\top \rho_d(h_d(x,0)) = 0\ \right\} \tag{19}$$
   for $\mathcal{H}_0$,
   - 3.b) every complete solution ϕ to $\mathcal{H}_0$ is Zeno,

then $\mathcal{A}$ is 0-input pre-asymptotically stable for $\mathcal{H}$.

4) flow-strict passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$, and 2.b) holds, then $\mathcal{A}$ is 0-input pre-asymptotically stable for $\mathcal{H}$.
5) jump-strict passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$, and 3.b) holds, then $\mathcal{A}$ is 0-input pre-asymptotically stable for $\mathcal{H}$.

IV. Passivity-based Control

The concepts of flow- and jump-passivity introduced in Definition 1 can be combined with the notion of detectability introduced in Section III-B and the properties of the solution given in Proposition 2 for stabilization by means of static output feedback. The result given in the following theorem, in particular, allows to directly employ passivity-based control paradigms – see for instance [4], [7] – for the special cases of flow and jump passivity in hybrid systems.

Theorem 1: Given a compact set $\mathcal{A} \subset \mathbb{R}^n$ and a hybrid system $\mathcal{H}$ satisfying (A1)-(A3) with continuous output maps $x \mapsto h_c(x)$ and $x \mapsto h_d(x)$ the following hold:

1) If $\mathcal{H}$ is flow-passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$ and there exists a continuous function $k_c : \mathbb{R}^{m_c} \to \mathbb{R}^{m_c}$, with $y_c^\top k_c(y_c) > 0$ for all $y_c \neq 0$ having defined $y_c = h_c(x)$, such that the resulting closed-loop system with $v_c = -k_c(y_c)$ and $v_d = 0$ has the following properties:
   1.1) the distance to $\mathcal{A}$ is detectable relative to
   $$\left\{x \;:\; h_c(x)^\top k_c(h_c(x)) = 0,\, (x, -k_c(h_c(x))) \in C \;\right\} \tag{20}$$
   with $v_d = 0$,
   1.2) every complete solution ϕ with $v_d = 0$ is such that for some $\delta > 0$ and some $J \in \mathbb{N}$ we have $t_{j+1} - t_j \geq \delta$ for all $j \geq J$,

   then the control law $v_c = -k_c(y_c)$, $v_d = 0$ renders $\mathcal{A}$ pre-asymptotically stable.
2) If $\mathcal{H}$ is jump-passive with respect to $\mathcal{A}$ with a storage function V that is positive definite with respect to $\mathcal{A}$ and there exists a continuous function $k_d : \mathbb{R}^{m_d} \to \mathbb{R}^{m_d}$, with $y_d^\top k_d(y_d) > 0$ for all $y_d \neq 0$ having defined $y_d = h_d(x)$, such that the resulting closed-loop system with $v_c = 0$ and $v_d = -k_d(y_d)$ has the following properties:
   2.1) the distance to $\mathcal{A}$ is detectable relative to
   $$\left\{x \;:\; h_d(x)^\top k_d(h_d(x)) = 0,\, (x, -k_d(h_d(x))) \in D \;\right\} \tag{21}$$
   with $v_c = 0$,
   2.2) every complete solution ϕ with $v_c = 0$ is Zeno

   then the control law $v_d = -k_d(y_d)$, $v_c = 0$ renders $\mathcal{A}$ pre-asymptotically stable.

For the proof of Theorem 1, the reader is referred to [22].

Remark. Theorem 1 extends the classical passivity control results (see for instance [2], [3], [1], [4]) to the class of hybrid systems considered in this work. With respect to other existing approaches available in literature, such the the ones in [16] for impulsive dynamical systems, the proposed framework here focuses also on the special cases of flow and jump passivity which have been shown to be relevant in some applications. In fact, the results in [16] cannot be applied to the application considered in this paper since the output strict passivity property does not hold both along flows and jumps. The approach proposed here links passivity to asymptotic stability thought detectability and, for the special cases, it requires also some properties of the solutions. The detectability conditions required here are weaker than the observability property imposed in [16]. ◁

A. Application re-revisited

Consider the hybrid system $\mathcal{H}_S$ given in Section III-A.1. The control goal is to stabilize the point-mass to a position in contact with the vertical surface, namely to render the set $\mathcal{A} = (x_1^\star, 0)$, with $x_1^\star \geq 0$, globally asymptotically stable for the closed-loop hybrid system. Theorem 1 can be employed to assert that property by means of the energy-based controller (11) (passivation by feedback and energy shaping) in which the remaining control input w_c is synthesized as a damping injection. This fact is established by the following proposition for which a proof is available in [22].

Proposition 3: For the hybrid system (9) with control input v_c chosen as in (11), the control law $w_c = -k_1 y_c$, with $k_1 > 0$, renders the compact set $\mathcal{A} = (x_1^\star, 0)$ globally asymptotically stable.

Remark. Observe that asymptotically the control input v_c in (11) is given by $v_c^\star(x_1^\star, 0) = k_c x_1^\star$. From a physical viewpoint, the mass is then applying a force to the vertical surface that can be varied according to the choice of the set-point position $x_1^\star \geq 0$. Passivity-based control techniques are in fact employed in several force control schemes (see [25] and references therein). ◁

V. Simulations

Taking advantage of the framework for numerical simulations of hybrid systems available at [26], this section presents some numerical results obtained considering the passivity-based control law derived in Section IV for the mechanical system described respectively in Sections II and III-A.1. The parameters of the system and of the passivity-based control law used in the simulations are $M = 1\ kg$, $\varrho = 1$, $k_c = 8\ N/m$, $b_c = 10\ Ns/m$, $k_P = 10$, $k_1 = 2$, $\bar{x}_2 = 0.1\ m/s$ and $x_1^\star = 0.1\ m$.

By considering as initial condition for the mass a certain constant distance from the vertical surface, in particular $x(0,0) = (1,0)$, for the position x_1 and the velocity x_2 we obtained the trajectories depicted respectively in Figures 2 and 3. Observe that at $t = 0$, $j = 0$ the mass, governed by the passivity-based control law (11) with $w_c = -k_1 y_c$, starts accelerating towards the surface. Then at $t \approx 0.5\ sec$ the surface is reached with a velocity larger than $\bar{x}_2$. Accordingly, the mass instantaneously rebounds subject to the jump map in (9). After the collision, the ball continues to flow until another rebound occurs. It is worth to note that, since during the continuous-time evolution the controller is dissipating kinetic energy, collisions are achieved with progressively decreasing impact velocities. As a consequence, once collisions are achieved with a speed lower or equal than $\bar{x}_2$, the impacts become compliant and the mass finally remains in contact with the surface reaching asymptotically the final desired position $x_1^\star = 0.1\ m$ by flowing only.

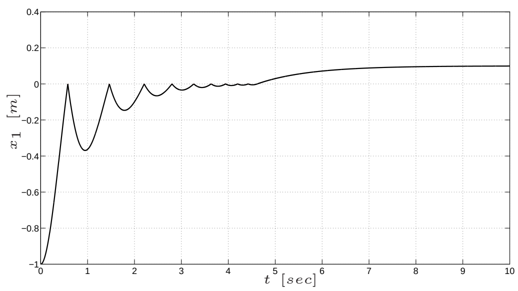

Fig. 2. Position x_1 of the mass during a simulation.

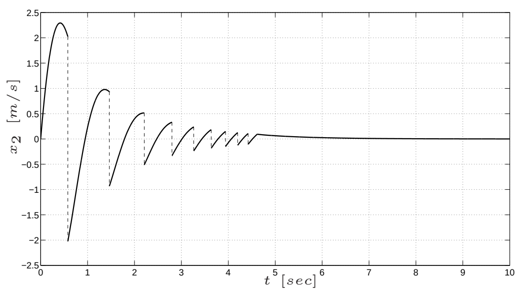

Fig. 3. Velocity x_2 of the point mass during a simulation.

VI. Conclusion

In this paper we considered the design of passivity-based controllers for a class of hybrid systems. Motivated by an application of a mechanical systems interacting with the surrounding environment, a weak notion of passivity, for systems in which dissipation of energy is allowed to happen only during the continuous or the discrete time behavior respectively, has been proposed and linked, through detectability, to asymptotic stability. The proposed methodology was employed to show the effectiveness of classical passivity-based control design in the application of interest. Future work will be focused on showing the effectiveness of passivity-based control paradigms for aerial vehicles physically interacting with the surrounding environment.

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