A Robust Hybrid Control Algorithm for a Single-Phase DC/AC Inverter with Variable Input Voltage

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***Abstract*— In this paper, we analyze the properties of the vector fields associated with all possible configurations of a single-phase DC/AC inverter with the objective of designing a hybrid controller for the generation of an approximation of a sinusoidal reference signal. Using forward invariance tools for general hybrid systems, a hybrid controller is designed for the switched differential equations capturing the dynamics of the DC/AC inverter. Then, global asymptotic stability of a set of points nearby the reference trajectory, called the tracking band, is established. This property is found to be robust to small perturbations, and variation of the input voltage. Simulations illustrating the major results are included.**

I. INTRODUCTION

Besides fossil and nuclear-based power, future energy distribution systems ought to be capable of interconnecting diverse renewable sources, such as hydroelectric generators, photovoltaic arrays, and wind turbines, as well as energy storage systems. A particular challenge imposed by these "smart grid" futuristic views is the high variability of the power provided by the renewable sources, mainly due to their high dependence on environmental conditions. In turn, this variability imposes a challenge to power conversion, in particular, between DC and AC signals.

In this paper, a single-phase DC/AC inverter, one of the most common topologies used in power conversion, is studied. This circuit is capable of transforming DC input voltage into an approximate AC output voltage. As shown in Figure 1, by controlling the positions of the four switches of the inverter, the sign of the input DC voltage to the RLC filter changes, and when appropriately controlled, the voltage across the capacitor and the current though the inductor can evolve almost sinusoidally. Typically, DC/AC inverters are controlled using Pulse Width Modulation (PWM) techniques. PWM-based controllers trigger switches of the inverters whenever the difference of a carrier signal, usually a triangular wave, and the reference sinusoidal signal changes sign. The performance of PWM-based controllers has been thoroughly studied in the literature [1], [2], [3]. One of the shortcomings of PWM-based controllers is that the control of the output voltage magnitude is not robust to changes of the input DC voltage. Without a DC voltage regulator at its input, the "sinusoidal" output would be significantly affected. Another disadvantage of PWM-based controllers is relative high harmonic distortions. This kind of critical issues in power conversion has led to the development of new control algorithms relying on recent advances of the theory of switching and hybrid systems [4], [5], [6].

In this paper, we propose a hybrid controller for a single phase DC/AC inverter designed using hybrid system theory. The proposed control law manages to accomplish the task of getting a sinusoidal output signal as close as possible to the reference signal by controlling the four switches. For this purpose, we use a switched differential equation to capture the dynamics of the DC/AC inverter. The proposed controller triggers switches based on the value of the current and voltage of the RLC filter. Results on forward invariance of sets for general hybrid systems are used to analyze the effect of the proposed controller. More precisely, we show that our controller renders a region around the reference trajectory, which we refer to as the tracking band, forward invariant and that solutions from outside this region converge to it in finite time. This property allows us to show global asymptotic stability of the tracking band, which, in turn, implies robustness to small perturbations and variation of the input voltage. Additionally, the harmonic distortion introduced by our controller is small according to our FFT analysis (and it appears to outperform a PWM-based controller).

The structure of this paper is as follows. After modeling the DC/AC inverter, basic concepts of hybrid systems are presented in Section III. Then, in Section IV-A, we introduce the reference trajectory, a ellipse-shaped limit cycle, on the current-voltage plane. The proposed control law is introduced in Sections IV-B. The properties of closed-loop system with our controller are studied in Sections V. Finally, in Section VI, simulations are presented to show the capabilities of the proposed controller.

II. MODELING A SINGLE-PHASE DC/AC INVERTER

A single-phase DC/AC inverter circuit consists of four controlled switches connecting to a series RLC filter, as shown in Figure 1.The DC signal V_{DC} is the input signal to the inverter. The output signal v_C denotes the voltage across the capacitor C, and i_L denotes the current through the inductor L. The objective of a controller selecting the positions of the switches $S_1 - S_4$ is to generate an output v_C that approximates a sinusoidal signal of a desired frequency by appropriately toggling the switches.

The presence of switches in the circuit introduces non-smooth dynamics. By controlling the position of the switches, to either "ON" or "OFF" position, the voltage V_{in}

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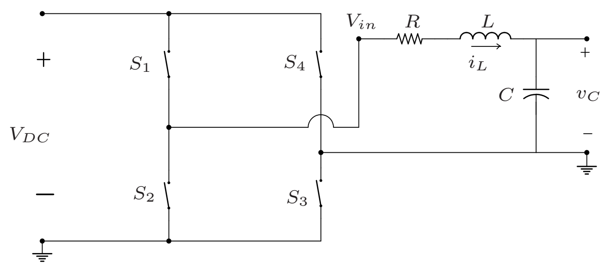


Fig. 1: Single-phase DC/AC inverter circuit diagram.

to the RLC filter will equal either V_{DC} , $-V_{DC}$, or 0. Differential equations describing the system dynamics are given as

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = f_q(z) := \begin{bmatrix} \frac{V_{DC}}{L}q - \frac{R}{L}i_L - \frac{1}{L}v_C \\ \frac{1}{C}i_L \end{bmatrix}, \tag{1}$$

where R, L, C are parameters of the circuit, $z := (i_L, v_C) \in \mathbb{R}^2$, and q is a logic variable that describes the position of the switches. In this way, $q \in Q := \{-1, 0, 1\}$ leads to the following states of interest of the inverter circuit:

$$\dot{i}_L = \begin{cases} \frac{V_{DC}}{L} - \frac{R}{L}i_L - \frac{1}{L}v_C & \text{when } S_1 = S_3 = \text{ON and } S_2 = S_4 = \text{OFF;} \\ -\frac{V_{DC}}{L} - \frac{R}{L}i_L - \frac{1}{L}v_C & \text{when } S_1 = S_3 = \text{OFF and } S_2 = S_4 = \text{ON;} \\ -\frac{R}{L}i_L - \frac{1}{L}v_C & \text{when } S_1 = S_4 = \text{OFF and } S_2 = S_3 = \text{ON} \end{cases} \tag{2}$$

$$\dot{v}_C = \frac{1}{C}i_L$$

Typically, DC/AC inverters are controlled by an algorithm generating a switching profile following the so-called Pulse Width Modulation (PWM) technique. In this paper, motivated by the shortcomings of PWM-based control for inverters discussed in the introduction, a new control law is developed and analyzed through a hybrid control approach for the purpose of having robust control of the inverter. Next, we introduce basic concepts of hybrid systems and develop new supporting results in the next section.

III. Basic Concepts of Hybrid Systems

A hybrid system $\mathcal{H}$, or more precisely, a closed-loop system with a hybrid controller in our case, can be written as

$$\mathcal{H} \begin{cases} \dot{x} = f(x) & x \in \mathcal{C} \\ x^+ \in G(x) & x \in \mathcal{D}, \end{cases} \tag{3}$$

where $\mathcal{C}, f, \mathcal{D}$, and G represent the flow set, the flow map, the jump set, and the jump map, respectively. Solutions to (3) have continuous and/or discrete behavior depending on the system data $(\mathcal{C}, f, \mathcal{D}, G)$. Following [7], besides the usual time variable $t \in \mathbb{R}_{\geq 0}$, we consider the number of jumps, $j \in \mathbb{N} := \{0, 1, 2, ...\}$, as an independent variable. Thus, hybrid time is parametrized by (t, j). The domain of a solution to $\mathcal{H}$ is given by a hybrid time domain. A hybrid time domain is defined as a subset E of $\mathbb{R}_{\geq 0} \times \mathbb{N}$ that, for each $(T, J) \in E$, $E \cap ([0, T] \times \{0, 1, ...J\})$ can be written as $\cup_{j=0}^{J-1} ([t_j, t_{j+1}], j)$ for some finite sequence of times $0 = t_0 \leq t_1 \leq t_2... \leq t_J$. A solution to the hybrid system (3) is given by a hybrid arc ϕ satisfying the dynamics of (3). A hybrid arc ϕ is a function on a hybrid time domain that, for each $j \in \mathbb{N}$, $t \mapsto \phi(t, j)$ is absolutely continuous on the interval $\{t : (t, j) \in \operatorname{dom} \phi\}$. A solution ϕ to (3) is said to be *complete* if $\operatorname{dom} \phi$ is unbounded and *maximal* if there does not exist another pair ϕ' such that ϕ is a truncation of ϕ' to some proper subset of $\operatorname{dom} \phi'$. Furthermore, we say that a set $K \subset \mathbb{R}^n$ is forward invariant for $\mathcal{H}$ if every maximal solution ϕ from K is complete and $\phi(t, j) \in K$ for all $(t, j) \in \operatorname{dom} \phi$. For more details about solutions to hybrid systems, see [7].

IV. A Hybrid Controller for the Generation of an Approximation of a Sinusoidal Voltage

A. *Sinusoidal Reference Trajectory*

Reference signals $t \mapsto (i_L^*(t), v_C^*(t))$ are given by the steady-state response of the RLC filter in Figure 1 to sinusoidal input signals $t \mapsto V_{in}(t) = A \sin(\omega t + \theta)$, where $A, \omega > 0$ are the magnitude and angular frequency, respectively, and θ is the initial phase. Using the equations of the filter, under the effect of the input $V_{in}(t)$, every steady-state solution, in particular, (i_L^*, v_C^*), satisfies $V(i_L^*(t), v_C^*(t)) = c$ for all $t \geq 0$, where

$$V(z) := \left(\frac{i_L}{a}\right)^2 + \left(\frac{v_C}{b}\right)^2 \quad z \in \mathbb{R}^2 \tag{4}$$

with constants a and b given by

$$a := \frac{1}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}}, \quad b := \frac{1}{C\omega\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}}.$$

B. *Control strategy*

A hybrid control strategy is developed for the inverter to switch among the three operation modes described in (2). This hybrid control strategy provides an alternative to the traditional PWM control approach with arbitrary precision for an inverter. More precisely, the hybrid control strategy guarantees that the output trajectory converges to a region (tracking band) nearby the reference trajectory satisfying (4).

1) Tracking Band: The tracking band is defined as a neighborhood around the set $\{z : V(z) = c\}$, which defines the reference trajectory. More precisely, given c_i and c_o such that $c_i < c < c_o$, the tracking band is given by

$$\{z \in \mathbb{R}^2 : c_i \leq V(z) \leq c_o\}. \tag{5}$$

On the (i_L, v_C) plane, the tracking band has an outer boundary given by $S_o = \{z \in \mathbb{R}^2 : V(z) = c_o\}$, which is the outer green dashed line in Figure 2, and an inner boundary given by $S_i = \{z \in \mathbb{R}^2 : V(z) = c_i\}$, which is the inner green dash line in Figure 2. The reference trajectory, which is the blue solid line in Figure 2, is enclosed by the tracking band. A trajectory to (2) with the proposed control strategy is shown in red solid line and, as the figure depicts, remains in the tracking band for all time while describing a “periodic” orbit. The parameters used for Figure 2 are: $R = 0.6\Omega$,

$L = 0.1$H, $C = 0.04$F, $V_{DC} = 5$V, $c_i = 0.9$, $c_o = 1.1$, and $c = 1$.

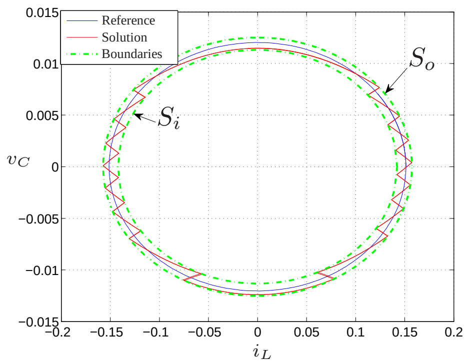


Fig. 2: A sample trajectory resulting from using the proposed control law.

2) Control Logic for Strong Forward Invariance of the Tracking Band: In Section IV-B.1, S_o, S_i, c_i, c_o were introduced to define the tracking band. Here, for the ease of introducing the control logic, we define a (small enough) positive parameter ϵ and the sets

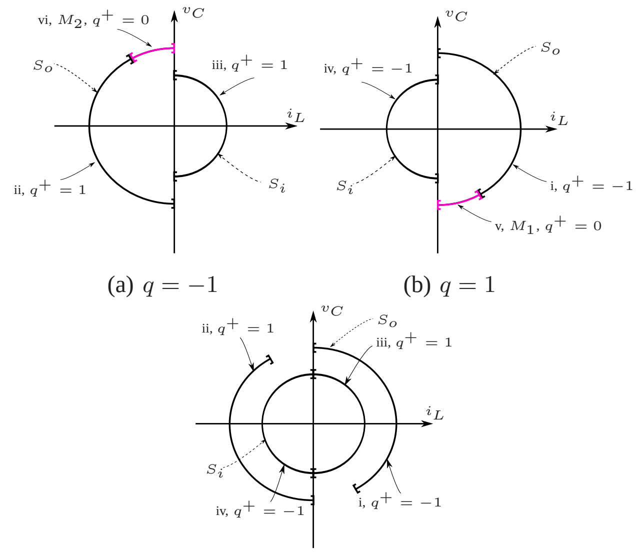
$$M_1 = \{z \in \mathbb{R}^2 : V(z) = c_o, 0 \le i_L \le \epsilon, v_C \le 0\};$$
$$M_2 = \{z \in \mathbb{R}^2 : V(z) = c_o, -\epsilon \le i_L \le 0, v_C \ge 0\}.$$

Proposed control algorithm for forward invariance: Using current values of z and q, switch q according to the following rules (see Figure 3):

i. if $z \in (S_o \setminus M_1) \cap \{z \in \mathbb{R}^2 : i_L \ge 0\}$ and $q \ne -1$, switch to vector field for $q = -1$ to steer the trajectory to S_i;
ii. if $z \in (S_o \setminus M_2) \cap \{z \in \mathbb{R}^2 : i_L \le 0\}$ and $q \ne 1$, switch to vector field for $q = 1$ to steer the trajectory to S_i;
iii. if $z \in S_i \cap \{z \in \mathbb{R}^2 : i_L \ge 0\}$ and $q = -1$ or $q = 0$, switch to vector field for $q = 1$ to steer the trajectory to S_o;
iv. if $z \in S_i \cap \{z \in \mathbb{R}^2 : i_L \le 0\}$ and $q = 1$ or $q = 0$, switch to vector field for $q = -1$ to steer the trajectory to S_o;
v. if $z \in M_1$ and $q = 1$, switch to vector field for $q = 0$ to steer the trajectory to the right hand side of the (i_L, v_C) plane;
vi. if $z \in M_2$ and $q = -1$, switch to vector field for $q = 0$ to steer the trajectory to the left hand side of the (i_L, v_C) plane.

Note that the proposed control algorithm includes regions M_1 and M_2, on which switches to mode $q = 0$ inside the tracking band take place. This mechanism is included to prevent fast switching at points in $\{z : V(z) = c_o, i_L = 0\}$, from where, when $q \in \{-1, 1\}$, solutions would flow "horizontally" (to the left or to the right) on the (i_L, v_C) plane.

Next, we propose a controller implementing the control logic described above, and state key properties of the vector fields for $q \in Q$.



(a) $q = -1$ (b) $q = 1$

(c) $q = 0$

Fig. 3: Regions on the (i_L, v_C) plane for each $q \in Q$ used in rules i-vi of the algorithm for forward invariance. The corresponding rules are also indicated.

A hybrid controller denoted by $\mathcal{H}_{\text{fw}} = (\mathcal{C}_{\text{fw}}, f_{\text{fw}}, \mathcal{D}_{\text{fw}}, G_{\text{fw}})$ is constructed based on the proposed control logic above. The controller has a logic state, which, with some abuse of notation, we denote as $q \in Q$, and has input z. Its dynamics are given by the hybrid system

$$\mathcal{H}_{\text{fw}} \begin{cases} \dot{q} = f_{\text{fw}}(q) & (q, z) \in \mathcal{C}_{\text{fw}} \\ q^+ \in G_{\text{fw}}(q) & (q, z) \in \mathcal{D}_{\text{fw}} \end{cases}$$

where the flow map f_{fw} is defined as

$$f_{\text{fw}}(q) := 0,$$

the flow set $\mathcal{C}_{\text{fw}}$ is defined as

$$\mathcal{C}_{\text{fw}} := \left\{(q, z) \in Q \times \mathbb{R}^2 : V(z) \in [c_i, c_o]\right\}, \qquad (6)$$

the jump map G_{fw} is defined as

$$G_{\text{fw}}(q) := \begin{cases} -1 & \text{if } q \ne -1 \text{ and} \\ & [(V(z) = c_o \text{ and } i_L \ge 0 \text{ and } z \notin M_1) \\ & \text{or } (V(z) = c_i \text{ and } i_L \le 0)]; \\ 0 & \text{if } (z \in M_1 \text{ and } i_L \ne \epsilon \text{ and } q = 1) \\ & \text{or } (z \in M_2 \text{ and } i_L \ne -\epsilon \text{ and } q = -1); \\ 1 & \text{if } q \ne 1 \text{ and} \\ & [(V(z) = c_o \text{ and } i_L \le 0 \text{ and } z \notin M_2) \\ & \text{or } (V(z) = c_i \text{ and } i_L \ge 0)]; \\ \{0, 1\} & \text{if } (V(z) = c_o, i_L = -\epsilon, v_C \ge 0); \\ \{-1, 0\} & \text{if } (V(z) = c_o, i_L = \epsilon, v_C \le 0); \end{cases}$$

and the jump set $\mathcal{D}_{\text{fw}}$ is defined as

$$\begin{aligned} \mathcal{D}_{\text{fw}} := \; & \{(q, z) \in Q \times \mathbb{R}^2 : V(z) = c_i, i_L q \le 0, q \ne 0\} \\ & \bigcup \{(q, z) \in Q \times \mathbb{R}^2 : V(z) = c_o, i_L q \ge 0, q \ne 0\} \\ & \bigcup \{(q, z) \in Q \times \mathbb{R}^2 : V(z) = c_i, q = 0\}. \end{aligned}$$

When $\mathcal{H}_{\text{fw}}$ is used to control the plant in (1), the output of the plant, which is z, becomes the input of $\mathcal{H}_{\text{fw}}$, and its output

q becomes the input of the plant. This yields a hybrid closed-loop system with state variable $\eta = [q \;\, z^\top]^\top$, which can be written as the hybrid system $\mathcal{H}^{cl}_{\text{fw}} = (\mathcal{C}_{\text{fw}}, f^{cl}_{\text{fw}}, \mathcal{D}_{\text{fw}}, G^{cl}_{\text{fw}})$ given by

$$\mathcal{H}^{cl}_{\text{fw}} \begin{cases} \dot{\eta} = f^{cl}_{\text{fw}}(\eta) & \eta \in \mathcal{C}_{\text{fw}} \\ \eta^+ \in G^{cl}_{\text{fw}}(\eta) & \eta \in \mathcal{D}_{\text{fw}}, \end{cases}$$

where

$$f^{cl}_{\text{fw}}(\eta) = \begin{bmatrix} 0 \\ \frac{V_{DC}}{L} q - \frac{R}{L} i_L - \frac{1}{L} v_C \\ \frac{1}{C} i_L \end{bmatrix}, G^{cl}_{\text{fw}}(\eta) = \begin{bmatrix} G_{\text{fw}}(q) \\ i_L \\ v_C \end{bmatrix}.$$

The closed-loop system $\mathcal{H}^{cl}_{\text{fw}}$ satisfies the hybrid basic conditions introduced in [7, Assumption 6.5].

Lemma 1 (Hybrid basic conditions): The hybrid model $\mathcal{H}^{cl}_{\text{fw}}$ of the inverter system satisfies the basic hybrid conditions, i.e., its data $(\mathcal{C}_{\text{fw}}, f^{cl}_{\text{fw}}, \mathcal{D}_{\text{fw}}, G^{cl}_{\text{fw}})$ is such that

(A1) $\mathcal{C}_{\text{fw}}$ and $\mathcal{D}_{\text{fw}}$ are closed sets;
(A2) $f^{cl}_{\text{fw}} : Q \times \mathbb{R}^2 \to Q \times \mathbb{R}^2$ is continuous;
(A3) $G^{cl}_{\text{fw}} : Q \times \mathbb{R}^2 \rightrightarrows Q \times \mathbb{R}^2$ is outer semicontinuous and locally bounded relative to $\mathcal{D}_{\text{fw}}$, and $\mathcal{D}_{\text{fw}} \subset \operatorname{dom} G^{cl}_{\text{fw}}$.

Thus, according to [7, Section 6.1], the hybrid closed-loop system $\mathcal{H}^{cl}_{\text{fw}}$ is a well-posed hybrid system. Then, the behavior of the hybrid system $\mathcal{H}^{cl}_{\text{fw}}$ is robust to small perturbations.

Lemma 2 (Inner product properties): Given positive system constants R, L, C, ω, V_{DC} such that $LC\omega^2 \geq 1$, the following hold:

a) $\langle \nabla V(z), f_q(z) \rangle \leq 0$, for all $(q, z) \in Q \times \mathbb{R}^2$ such that $z \in \Gamma$, $(q, i_L) \in \{(q, i_L) \in Q \times \mathbb{R} : i_L \leq 0, q = 1\} \cup \{(q, i_L) \in Q \times \mathbb{R} : i_L \geq 0, q = -1\}$;
b) $\langle \nabla V(z), f_q(z) \rangle \geq 0$, for all $(q, z \in Q \times \mathbb{R}^2)$ such that $z \in \Gamma$, $(q, i_L) \in \{(q, i_L) \in Q \times \mathbb{R} : i_L \leq 0, q = -1\} \cup \{(q, i_L) \in Q \times \mathbb{R} : i_L \geq 0, q = 1\}$;
c) $\langle \nabla V(z), f_q(z) \rangle \leq 0$, for all $(q, z) \in Q \times \mathbb{R}^2$ such that $z \in M_1 \cup M_2$ and $q = 0$;

where Γ is defined as

$$\Gamma = \{z \in \mathbb{R}^2 : -\alpha V_{DC} \leq -\alpha R i_L + (\beta - \alpha) v_C \leq \alpha V_{DC}\}$$

with $\alpha = \dfrac{2}{a^2 L}$ and $\beta = \dfrac{2}{b^2 C}$.

Define the set $\mathcal{T} := Q \times \{z \in \mathbb{R}^2 : V(z) \in [c_1, c_o]\}$, i.e., points in $\mathcal{T}$ are located in the tracking band on the (i_L, v_C) plane, and have state $q \in Q$. The following result states that the set $\mathcal{T}$ is a forward invariant set for the closed-loop system $\mathcal{H}^{cl}_{\text{fw}}$.

Proposition 1 (Forward invariance): Given positive system constants R, L, C, ω, V_{DC} such that $LC\omega^2 > 1$, and $c_i < c_o$ such that $\mathcal{T} \subset Q \times \Gamma$ (see Lemma 2 for the definition of Γ), $\mathcal{T}$ is forward invariant for the hybrid closed-loop system $\mathcal{H}^{cl}_{\text{fw}} = (\mathcal{C}_{\text{fw}}, f^{cl}_{\text{fw}}, \mathcal{D}_{\text{fw}}, G^{cl}_{\text{fw}})$.

Proposition 1 implies that all solutions to the hybrid closed-loop system $\mathcal{H}^{cl}_{\text{fw}}$ stay in $\mathcal{T}$. Within $\mathcal{T}$, the solutions to $\mathcal{H}^{cl}_{\text{fw}}$ evolve counterclockwise due to the direction of the vector fields. This property follows directly from the closed-loop flow map f^{cl}_{fw}.

3) Augmented Logic for Global Convergence: Global convergence to $\mathcal{T}$ can be guaranteed by adding a controller that steers solutions into $\mathcal{T}$ globally. To obtain such a property, a controller that guarantees the following is required:

1) solutions from every point outside of S_o converge to S_o in finite time;
2) solutions from every point inside of S_i converge to S_i in finite time.

In this section, we introduce one possible controller that guarantees the global convergence property.

The controller is defined as follows. When z is outside the (interior of) the tracking band (5), we use a static controller $\mathcal{H}_{\text{g}}$ defined on

$$\mathcal{C}_{\text{g}} := \{z \in \mathbb{R}^2 : V(z) \geq c_o\} \bigcup \{z \in \mathbb{R}^2 : V(z) \leq c_i\}$$

and given by

$$\kappa(z) := \begin{cases} 0 & \text{if } V(z) \geq c_o \\ m & \text{if } V(z) \leq c_i, \end{cases}$$

where m is a constant parameter taking value from $\{-1, 1\}$. The static feedback law κ is the output of $\mathcal{H}_{\text{g}}$, which is used to control q of the plant (1), while its input is the current and voltage vector z. In this way, the choice $\kappa = 0$, which selects the vector field $f_0(z)$ of (1), is used to steer the solutions to S_o from outside of $\{z \in \mathbb{R}^2 : V(z) < c_o\}$. The choice $\kappa = -1$ (or $\kappa = 1$ depending on the value of m) is used to steer the solution to S_i from inside $\{z \in \mathbb{R}^2 : V(z) > c_i\}$.

Now, similar to $\mathcal{H}^{cl}_{\text{fw}}$, we define a closed-loop system $\mathcal{H}^{cl}_{\text{g}}$ by applying controller $\mathcal{H}_{\text{g}}$ to the plant (1). The system $\mathcal{H}^{cl}_{\text{g}}$ has state z and is defined on $\mathcal{C}_{\text{g}}$. The dynamics of $\mathcal{H}^{cl}_{\text{g}}$ are given by

$$\dot{z} = f_{\text{g}}(z) := f_{\kappa(z)}(z) \qquad z \in \mathcal{C}_{\text{g}}.$$

Proposition 2 (Global Convergence): Given positive system constants R, L, C, ω, V_{DC} and $c_i < c_o$ such that $LC\omega^2 > 1$ and $V_{DC} > b\sqrt{c_o}$, from every point z such that $V(z) \leq c_i$ or $V(z) \geq c_o$, the unique solution to $\mathcal{H}^{cl}_{\text{g}}$ converges to the tracking band (5) in finite time.

4) Supervisor Controller: With appropriately chosen parameters for controllers $\mathcal{H}_{\text{fw}}$ and $\mathcal{H}_{\text{g}}$, we can globally "track" any reference trajectory (i^*_L, v^*_C) described by (4). For this purpose, we introduce a hybrid supervisor controller denoted $\mathcal{H}_{\text{s}}$ that uses information of the location of z and switches between controller $\mathcal{H}_{\text{fw}}$ and $\mathcal{H}_{\text{g}}$ to guarantee global convergence and forward invariance of the tracking band. Figure 4 shows the feedback control architecture.

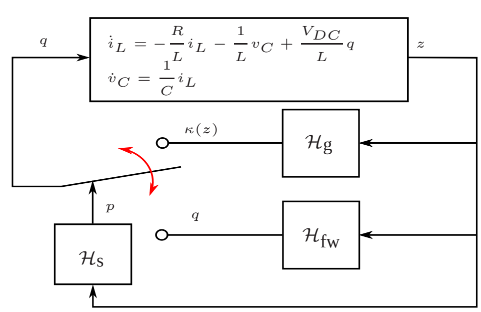


Fig. 4: Full closed-loop system with $\mathcal{H}_{\text{s}}, \mathcal{H}_{\text{g}}$, and $\mathcal{H}_{\text{fw}}$.

The supervisor $\mathcal{H}_s = (\mathcal{C}_s, f_s, \mathcal{D}_s, g_s)$ has state p and input z. The state variable p takes values from $P := \{1, 2\}$, which denotes the following:

$$p = \begin{cases} 1 & \text{indicates that controller } \mathcal{H}_{\text{fw}} \text{ is in the loop} \\ 2 & \text{indicates that controller } \mathcal{H}_{\text{g}} \text{ is in the loop.} \end{cases}$$

The dynamics of the hybrid controller $\mathcal{H}_s$ can be described as

$$\mathcal{H}_s \begin{cases} \dot{p} = f_s(p) & (p, z) \in \mathcal{C}_s \\ p^+ = g_s(p) & (p, z) \in \mathcal{D}_s \end{cases}$$

with flow map given by $f_s(p) := 0$, flow set defined as

$$\begin{aligned} \mathcal{C}_s :=& \{(p, z) \in P \times \mathbb{R}^2 : V(z) \in [c_1, c_o], p = 1\} \\ & \bigcup \{(p, z) \in P \times \mathbb{R}^2 : V(z) \geq c_o, p = 2\} \\ & \bigcup \{(p, z) \in P \times \mathbb{R}^2 : V(z) \leq c_i, p = 2\}, \end{aligned}$$

jump map given by $g_s(p) := 1$, and jump set defined as

$$\mathcal{D}_s := \{(p, z) \in \{1, 2\} \times \mathbb{R}^2 : c_i \leq V(z) \leq c_o, p = 2\}$$

Note that we constraint the definitions of $\mathcal{C}_s$ and $\mathcal{D}_s$ such that jumps from $p = 1$ to $p = 2$ are not allowed.[1]

V. Properties of the Full Closed-Loop System

In this section, the properties of full closed-loop system $\mathcal{H}$ that combines the dynamics of three controllers, $\mathcal{H}_{\text{fw}}$, $\mathcal{H}_{\text{g}}$ and $\mathcal{H}_s$, are analyzed. The closed-loop system is autonomous and has state variable $x = [p \; q \; z^\top]^\top$. Its hybrid model is given by

$$\mathcal{H} \begin{cases} \dot{x} = f(x) & x \in \mathcal{C} \\ x^+ \in G(x) & x \in \mathcal{D} \end{cases} \tag{7}$$

where the flow map f is given as

$$f(x) = \begin{bmatrix} 0 \\ 0 \\ -\frac{R}{L} i_L - \frac{1}{L} v_C + \frac{V_{DC}}{L} q \\ \frac{1}{C} i_L \end{bmatrix},$$

the flow set $\mathcal{C}$ is given as [2]

$$\begin{aligned} \mathcal{C} =& \{x \in P \times Q \times \mathbb{R}^2 : p = 1, (q, z) \in \mathcal{C}_{\text{fw}}\} \\ & \bigcup \{x \in P \times Q \times \mathbb{R}^2 : p = 2, z \in \mathcal{C}_{\text{g}}\}, \end{aligned}$$

the jump map is given as

$$G(x) = \begin{bmatrix} 1 \\ G_{\text{fw}}(q) \\ i_L \\ v_C \end{bmatrix},$$

and the jump set is given as

$$\begin{aligned} \mathcal{D} =& \left\{x \in P \times Q \times \mathbb{R}^2 : (p, z) \in \mathcal{D}_s\right\} \\ & \bigcup \left\{x \in P \times Q \times \mathbb{R}^2 : p = 1, (q, z) \in \mathcal{D}_{\text{fw}}\right\} \\ & \bigcup \left\{x \in P \times Q \times \mathbb{R}^2 : p = 2, z \in \mathcal{D}_{\text{g}}\right\}. \end{aligned}$$

[1] If we allow jumps from $p = 1$ to $p = 2$, there would appear Zeno solutions on the boundaries of the tracking band $\mathcal{T}$.

[2] Note that $\mathcal{C}_s$ is not part of the definition of $\mathcal{C}$ since, by the definition of $\mathcal{C}_{\text{fw}}$ and $\mathcal{C}_{\text{g}}, x \in \mathcal{C}$ if and only if $(p, z) \in \mathcal{C}_s$.

Using the fact that the closed-loop system $\mathcal{H}$ is a well-posed hybrid system, we can show global asymptotic stability of the tracking band $\mathcal{T}$.

Theorem 1 (Global asymptotic stability): Given a desired reference trajectory (4), for the hybrid system $\mathcal{H}$ in (7) with positive system parameters R, L, C, ω, V_{DC} and $c_i < c_o$ such that $LC\omega^2 > 1$ and $V_{DC} > b\sqrt{c_o}$, the (compact) set $\mathcal{T}$ is globally asymptotically stable for $\mathcal{H}$.

The result in Theorem 1 implies that our controller is robust to variations in the input voltage V_{DC}. In fact, when V_{DC} varies and remains in the range $(b\sqrt{c_o}, \infty)$, the controller is capable of steering the trajectory to the tracking band and render it forward invariant. Moreover, since the closed-loop system $\mathcal{H}$ satisfies the hybrid basic conditions in [7], the stability property is robust, in particular, to small measurement noise and unmodeled dynamics.

VI. Simulation Results

In this section, we show simulation results to highlight the features of full closed-loop hybrid system. All simulations are implemented in the Hybrid Equations (HyEQ) Toolbox via Simulink (see [8]). Unless stated otherwise, all simulations have the following system constants: $R = 0.6\Omega, L = 0.1H, C = 0.04F, \omega = 100\pi$, $V_{DC} = 5V, \epsilon = 0.05, c_o = 1.1, c_i = 0.9, a = 0.15$, and $c = 1$.

A. Simulations of closed-loop system $\mathcal{H}_{fw}^{cl}$

1) Simulation results of the closed-loop system $\mathcal{H}_{\text{fw}}^{cl}$ with initial location of z inside the interior of tracking band at $(0.1, 0.009)$ and initial q given by $q_0 \in Q$ are shown in Figure 5. As shown, with the same initial z at $(0.1, 0.009)$, for each possible initial logic variable value $q \in Q$, the solution to $\mathcal{H}_{\text{fw}}^{cl}$ stays inside the tracking band.

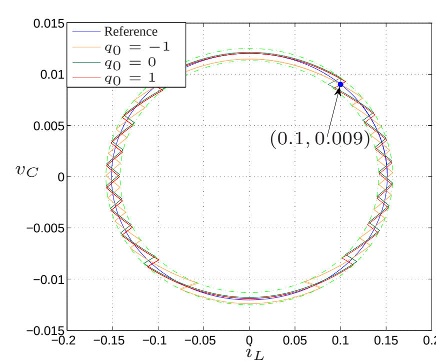
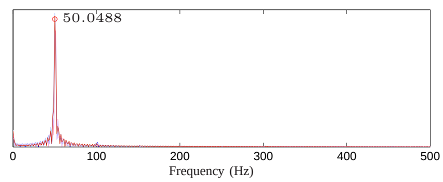


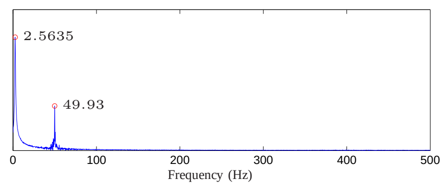
Fig. 5: Simulations of $\mathcal{H}_{\text{fw}}^{cl}$ with initial $z = (0.1, 0.009)$, and different initial values of q.

2) The FFT for the signal $t \mapsto v_c(t, \cdot)$ for the given set of system parameters set with 4 random initial conditions and $z = (0.1, 0.009)$ are presented in Figure 6a. As shown, the peak frequencies are at 50.0488 Hz, which is quite close to the reference frequency of 50 Hz. The harmonic distortion introduced by our controller is relatively small, when compared to the FFT of the same signal for a PWM-based controller, see Figure 6b. The

PWM controller in Figure 6b is a double sided PWM controller, which has a triangular shape carrier and a sinusoidal reference signal. Note that the spectrum for the latter is much richer for small frequency values.



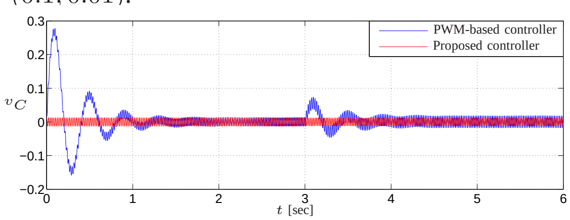
(a) with proposed $\mathcal{H}^{cl}_{\text{fw}}$.



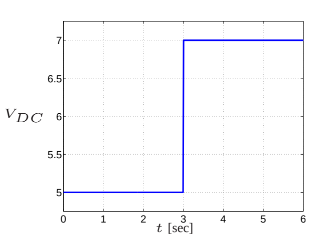
(b) with PWM-based controller.

Fig. 6: FFT of v_C output obtained by different controllers.

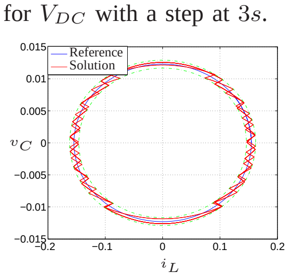
3) This simulation confirms that the proposed controller is robust to variations of V_{DC}, which is a key robustness property of our controller when compare to a PWM-based controller. Figure 7a shows steady v_C output of the inverter with the proposed controller (in red), even when there is a step change in the value of V_{DC}, see Figure 7b. On the other hand, the v_C output of the inverter with a PWM-based controller (in blue) has significant transient response and enlarged magnitude after the step in V_{DC} from $5V$ to $7V$ at $3s$. Both solutions have the same initial condition $z(0,0) = (0.1, 0.01)$.



(a) v_C output of the single-phase inverter with PWM-based and proposed hybrid controllers for V_{DC} with a step at $3s$.



(b) V_{DC} signal.

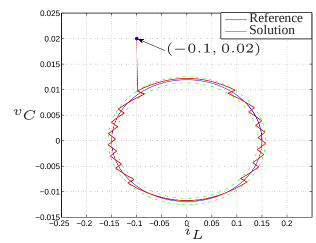


(c) Corresponding trajectory of proposed controller.

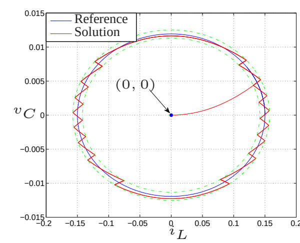
Fig. 7: Simulations with a step in V_{DC} at $3s$.

B. Simulations of full closed-loop system $\mathcal{H}$

For the full closed-loop system $\mathcal{H}$, which employs the supervisor $\mathcal{H}_{\text{s}}$, we show simulations with different initial conditions. A simulation of $\mathcal{H}$ with initial condition $x_0 = (p_0, q_0, z_0) = (2, 1, -0.1, 0.02)$, which is outside S_o, is shown in Figure 8a. The solution starts from the outside of S_o, $\mathcal{H}_{\text{s}}$ keeps p at 2, and the solution flows with the vector field for $q = \kappa(z) = 0$ until it hits S_o. Then, p is switched to 1 by $\mathcal{H}_{\text{s}}$, and the switching mechanism of $\mathcal{H}_{\text{fw}}$ is used to keep the solution within the tracking band from then on.



(a) Solution to initial condition outside S_o.



(b) Solution to initial condition inside S_i.

Fig. 8: Simulations of full controller setup when initial condition is outside $\mathcal{T}$.

A simulation of $\mathcal{H}$ with initial condition x_0 inside S_i is shown in Figure 8b. The solution starts with $p = 2$. Then, p is switched to 1 by $\mathcal{H}_{\text{s}}$, and the solution stays within the tracking band from then on.

VII. Conclusion

In this paper, a hybrid controller for a single-phase DC/AC inverter has been designed. Given appropriate system constants and reference signal, the proposed hybrid controller is robust to variable input voltage and small perturbations, while guaranteeing global convergence to the forward invariant tracking band in finite time. Numerical results show that the output voltage has less harmonic distortion than the output of PWM-based control technique.

References

[1] J. Rodriguez, J. Pontt, C. A. Silva, P. Correa, P. Lezana, P Cortés, and U. Ammann. Predictive current control of a voltage source inverter. *IEEE Transactions on Industrial Electronics,*, 54(1):495–503, 2007.

[2] J. Rodriguez, J.-S. Lai, and F. Zheng P. Multilevel inverters: a survey of topologies, controls, and applications. *IEEE Transactions onIndustrial Electronics,*, 49(4):724–738, 2002.

[3] S. Mariéthoz and M. Morari. Explicit model-predictive control of a PWM inverter with an LCL filter. *Industrial Electronics, IEEE Transactions on*, 56(2):389–399, 2009.

[4] P.C. Loh, G.H. Bode, and P.-C. Tan. Modular hysteresis current control of hybrid multilevel inverters. 152(1):1–8, 2005.

[5] M. Senesky, G. Eirea, and T. J. Koo. Hybrid modelling and control of power electronics. In *Hybrid Systems: Computation and Control*, pages 450–465. Springer, 2003.

[6] S. Kouro, P. Cortés, R. Vargas, U. Ammann, and J. Rodríguez. Model predictive controla simple and powerful method to control power converters. *IEEE Transactions on Industrial Electronics,*, 56(6):1826–1838, 2009.

[7] A. R. Teel R. Goebel, R. G. Sanfelice. *Hybrid Dynamical Systems*. Princeton University Press, 2012.

[8] R. G. Sanfelice, D. A. Copp, and P. Nanez. A toolbox for simulation of hybrid systems in Matlab/Simulink: Hybrid Equations (HyEQ) Toolbox. In *Proceedings of Hybrid Systems: Computation and Control Conference*, pages 101–106, 2013.